A Short Course on Boundary Integral Equations

Four Lectures on Tuesday 14th September as Part of the 3rd International Conference on Boundary Integral Equations: Theory and Applications

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The four talks are based on my book, "Strongly Elliptic Systems and Boundary Integral Equations" (Cambridge, 2000)[1], and are intended for graduate students in computational PDE with an interest in boundary integral methods. I will assume some prior exposure to Sobolev spaces and weak solutions, such as a first course in the mathematics of finite elements, and obviously it would be useful to have seen a boundary integral equation before. The talks aim to give an overview of key results needed for the analysis of boundary element methods, with at least an outline of some proofs. The first two talks cover standard background material in elliptic PDE whereas the remaining two talks are specific to boundary integral equations. Throughout, the results and techniques are valid for Lipschitz domains.

1. **Elliptic PDEs and Sobolev Spaces**

We introduce a class of second-order partial differential equations that includes the Laplace equation and the Lame equations of linear elasticity as standard examples. The first Green identity gives rise to an associated sesquilinear form that in turn leads naturally to the notion of a Sobolev space. We define Sobolev spaces of arbitrary real order based on L_2 and describe their duality relations.

2. **Fredholm Property and Elliptic BVPs**

After explaining the trace concept and the generalized conormal derivative, we consider a simple class of abstract variational problem and then introduce the notion of strong ellipticity. With these prerequisites in place, existence and uniqueness results for the Dirichlet and Neumann problems are obtained in the form of the Fredholm Alternative.

3. **Surface Potentials**

We introduce the fundamental solution and see how this leads to the third Green identity, involving the usual volume and surface potentials. The focus of this talk is an outline of the proof of the jump relations and mapping properties for a Lipschitz domain.

4. **Boundary Integral Equations**

We see how the Dirichlet and Neumann problems may be reformulated as boundary integral equations of the first kind, and how these integral equations fall within the framework of the abstract variational problem in Lecture 2. We also discuss the integral formulation of an elliptic problem with mixed Dirichlet and Neumann boundary conditions.

[1] Currently stocked on Campus in Waterstones book shop!