

# $\mathcal{H}^2$ -matrices for High Frequency Helmholtz Equation

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**Abstract.** We investigate the numerical solution of the Helmholtz equation by the boundary element method (BEM). In such methods the boundary is subdivided into  $n$  elements and the problem is reduced to the solution of an  $n \times n$  linear system of equations. The matrices describing the system are dense and hence direct methods are prohibitively expensive for large  $n$ . Iterative methods have to be used and our aim is to reduce the cost of matrix-vector multiplication from  $O(n^2)$  to an almost linear complexity.

We are interested in the high frequency Helmholtz equation where in two dimensions the number of elements  $n$  is proportional to the wave number  $\kappa$ . We apply the  $\mathcal{H}^2$ -matrix approach to the two dimensional problem and obtain an algorithm with complexity  $O(n \log^2 n)$  for the construction of the matrix and for the matrix-vector multiplication. We also consider how algebraic compression of the  $\mathcal{H}^2$ -matrix representation can be used to optimize the time needed for matrix-vector multiplication.

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