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Computing with Singular and Nearly Singular Integrals

We will describe a simple, direct approach to computing a singular or nearly singular integral, such as a harmonic function given by a layer potential on a curve in 2D or a surface in 3D. The value is found by a standard quadrature, using a regularized form of the singularity, with correction terms added for the errors due to regularization and discretization. These corrections are found by local analysis near the singularity. This technique might be useful in viscous fluid calculations with moving interfaces, since a pressure term due to a boundary force can be written as a layer potential. The accurate evaluation of a layer potential near the curve or surface on which it is defined is not routine, since the integral is nearly singular. In work with M.-C. Lai, we solve boundary value problems in 2D by computing the integral at grid points near the curve as described and using these values to find those at all points. A similar approach works in 3D, with the surface integrals computed in overlapping coordinate grids on the surface. To solve a boundary value problem, we first need to solve an integral equation for the strength of a dipole layer on the surface. We have proved that the solution of the discrete integral equation converges to the exact solution. In related work with G. Baker, we use a special choice of regularization in a boundary integral calculation of an unstable interface in 2D inviscid flow, such as a Rayleigh-Taylor flow with a heavy fluid over a lighter fluid.