Ultra-Weak Variational Formulation and Integral Representation using a Fast Multipole Method for the Equations of Electromagnetism

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A volumetric numerical solution of the time-harmonic Maxwell's equations in an exterior domain always implies the difficult problem of an artificial boundary around the obstacle. An absorbing boundary condition on the artificial boundary needs this boundary to be sufficiently far from the obstacle. The discretized exterior domain is then large and implies a numerical solution with a large number of degrees of freedom. The idea introduced by C. Hazard and M. Lenoir ([2]) consists in considering an integral representation of the unknown on the artificial boundary given by the unknown on a third boundary Σ as close to the boundary of the obstacle as we wish under the condition that the region between the artificial boundary and Σ is homogeneous. Such an approximation enables one to take the artificial boundary very close to the obstacle.

We applied the integral representation to the Ultra-Weak Variational Formulation (UWVF) which is a volumetric numerical method developed by B. Després and O. Cessenat ([1]). The UWVF involves discontinuous basis functions which are derived from a set of plane waves. With respect to the choice of p, the number of basis functions per tetrahedron of the mesh, one can consider a mesh which is coarser than the one needed for a regular FEM. However, the number of tetrahedra is still of order of κ^3 where κ is the wave number. The cost of the solution is then of order of $\kappa^3 p^2$. When using an integral representation, the artificial boundary is considered quite close to the obstacle. The number of tetrahedra is then of order of κ^2. A combination of the UWVF and an integral representation implies a numerical solution with a cost of order of $\kappa^2 p^2 + \kappa^4 p^2$. The second term is relative to the integral representation. The use of a fast multipole method ([3]) is then a solution to reduce this second term.

By using a one level fast multipole method, the algorithm of the numerical solution has the complexity $\mathcal{O}(\kappa^2 p^2 + \kappa^3 p)$. Using a multilevel fast multipole method, the complexity reduces to $\kappa^2 p^2 + \kappa^2 \ln(\kappa) p$. Such a combination of numerical methods appears very interesting for its complexity. It has other relevant properties. When the artificial boundary and the third boundary Σ are not connected, the integral representation does not involve singularities. Moreover, with a good choice of the distance between these boundaries, one can consider a fast multipole method with no close interaction which involves the opportunity to consider local refinements with no lost of efficiency of the fast multipole method.

References

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