BOUNDARY INTEGRAL EQUATIONS ON NON–SMOOTH CURVES

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If we apply a potential method to a boundary value problem (BVP) for elliptic partial differential equation of second order in a plain domain $\Omega \subset \mathbb{R}^2$, we obtain boundary integral (pseudodifferential) equation (BIE) on the boundary $\Gamma = \partial\Omega$. Considering the Dirichlet problem for the Laplace operator as a model, we obtain the following model boundary integral equation

$$\pm\frac{1}{2}\varphi(t) + \frac{1}{2\pi i}\int_{\Gamma} \partial_{n(\tau)} \log|t-\tau|\varphi(\tau)|d\tau| = f(t)\,, \quad t \in \Gamma\,, \tag{1}$$

where $n(\tau)$ denotes the outer normal vector to the boundary curve Γ at the point $\tau \in \Gamma$ and $\partial_{n(\tau)}$ denotes the normal derivative.

If the boundary is smooth the reduction is equivalent and BIE is Fredholm in all classical function spaces (e..g., in Bessel potential $\mathbb{H}^s_p(\Gamma, \rho)$ with weight $\rho(t) = \prod_{j=1}^n |t - c_j|^{\alpha_j}$ for all $s \in \mathbb{R}$, $1 \leq p \leq \infty$ and $-1 < p\alpha_j < p - 1$.). If the boundary is piecewise–smooth or Lipschitz, we can not define invariantly the spaces with large parameter $|s| \geq 1 + 1/p$. Although the reduction is equivalent but BIEs are Fredholm in all classical function spaces except some discrete set of parameters of the spaces.

Here we suggest new spaces $\mathbb{KH}^s_p(\Gamma, \rho)$ which are defined on piecewise-smooth Γ for arbitrary $s \in \mathbb{R}$ and the Fredholm properties of equation (1) does not depend on s.

Even more dramatically changes the situation when the domain Ω has either outward or inward peaks (i.e., Γ has cusps). Although (1) is a convolution–type equation when cusps occur, the symbol of equation is degenerate.

Problem arises also with equivalence of BVP and of corresponding BIE in classical formulation. This requires investigations of equation (1) in spaces of functions which are subject to complicated additional restrictions.

Survey of recent results on the above mentioned topics will be presented.