A weakly-singular piecewise linear and quadratic collocation method for solving the exterior Helmholtz problem valid for all real and positive wavenumbers.

Paul J. Harris, University of Brighton,
Ke Chen, University of Liverpool,
Jin Cheng, Fudan University.

It is well known that by using the Burton and Miller method it is possible to reformulate the exterior Helmholtz problem as an integral equation which is valid for all real and positive values of the wavenumber. However, it is equally well known that this formulation introduces an integral operator with a hyper-singular kernel function which cannot be evaluated using conventional numerical techniques. This problem can be overcome by either using the Galerkin method with polynomial basis functions of any order to solve the integral equation, or by using the collocation method provided a piecewise constant approximation to the the solution is used. In either of these cases the hyper-singular integral can be transformed into a weakly-singular integral, but if the collocation method is used with non-constant basis functions then the hyper-singular operator is still a problem.

In this paper we present an analysis of the integral with the hyper-singular kernel function and show how it can be transformed into three integrals each of which is at worst weakly singular for polynomial basis functions of any order. We illustrate our method with a number of typical examples and show how the new method is considerably more accurate than the more usual piecewise constant method.