

# MATHEMATICAL PROBLEMS OF THE ELASTICITY THEORY OF CHIRAL MATERIALS

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A solid which is not isotropic with respect to inversion is called *chiral*, *noncentrosymmetric*, *acentric*, or *hemitropic*. Materials may exhibit chirality on the atomic scale, as in quartz and in biological molecules, as well as on a large scale, as in composites with helical or screw-shaped inclusions.

The main goal of our investigation is analysis of the basic boundary value and initial boundary value problems of the theory of hemitropic elasticity for bodies of arbitrary geometrical shape. We consider separately the equilibrium equations of statics, the steady state oscillation equations (the time harmonic dependent case), and the general equations of dynamics, and develop the potential method (boundary integral equations method) to obtain the uniqueness, existence, and regularity results.

To this end, the corresponding  $(6 \times 6)$  matrices of fundamental solutions are constructed explicitly (in terms of elementary functions) and the generalized Sommerfeld-Kupradze type radiation conditions are formulated which play a crucial role to establish the uniqueness results in the case of exterior boundary value problems (BVP).

The boundary integral operators generated by the single- and double-layer potentials are studied and their ellipticity and normal solvability properties are established for smooth and non-smooth (Lipschitz) domains. Due to the Lipschitz character of the boundary essential difficulties arise in the study of invertibility of the corresponding strongly singular boundary integral operators which, in general, are not classical pseudodifferential operators. Therefore, in this case one needs a special approach which is completely different from the case of smooth domains. For example, in our analysis we need the Rellich-Payne-Weinberegger type identity and some auxiliary estimates relating the tangential and normal derivatives of solutions.

Based on the results obtained, the uniqueness and existence theorems of solutions to the basic BVPs (in particular, the Dirichlet, Neumann and mixed problems) are proved by the layer potential method in various Hölder  $(C^{k,\alpha})$ , Sobolev-Slobodetski  $(W_p^s)$  and Besov  $(B_{p,q}^s)$  function spaces.