An approach to robust high frequency boundary integral method via symbol of DtN operator

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Consider the exterior Dirichlet problem for the Helmholtz equation outside a bounded domain $\Omega \in \mathbf{R}^2$ with smooth boundary Γ

$$u_{xx} + u_{yy} + k^2 u = 0, \qquad u|_\Gamma = f, \qquad \partial_r u - iku = o(r^{-1/2}). \qquad (1)$$

The Dirichlet-to-Neumann operator DtN : $f \to \partial_n u\,|_\Gamma$ associated with problem (1) depends on k. Using parametrization of Γ by the normalized arclength $s \bmod 2\pi$, we treat DtN(k) as a pseudodifferential operator of order 1 on the unit circle. Let $\sigma(s,n;\,k)$, $n \in \mathbf{Z}$, be its discrete symbol.

If Ω is a circle, then problem (1) has an explicit solution in terms of Hankel functions. The symbol $\sigma(s,n;\,k)$ admits the double asymptotics

$$\sigma(s,n;\,k) \;\sim\; i\sqrt{k^2-n^2}, \qquad k,n \to \infty, \;\; k/n \to \text{const}. \qquad (2)$$

The branch of the root satisfies $\mathrm{Re}\sqrt{} \ge 0$, $\mathrm{Im}\sqrt{} \ge 0$.

Conjecture. *Asymptotics* (2) *holds for any domain* Ω *with smooth boundary.*

An analogous statement in the case of non-real k is a rather simple theorem; proof is based on ellipticity with parameter of the Helmholtz operator. Our conjecture is backed by numerical experiments, including non-convex domains and gratings (with an appropriately modified radiation condition). It is consistent at the physical level of rigor with Kirchhoff approximation. Formula (2), unlike Kirchhoff's, isn't sensitive to the presence of flattened boundary regions.

The last property may be used to overcome at least part of complications inherent to usual treatments of the high-frequency problems, where even computational formulae depend on the order of tangency of primary and secondary rays to Γ.