A diffraction problem in the half space: Solving the forward and the inverse problem

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Abstract

Let $\Omega = \mathbb{R}^3_+$ be the upper half space in $\mathbb{R}^3$ and $\Gamma = \mathbb{R}^2$ its boundary. For a nonnegative scalar conductivity σ the electrostatic potential u inside Ω fulfills the PDE

$$\operatorname{div}(\sigma \operatorname{grad} u) = 0.$$

If a current density $f \neq 0$ is induced on the boundary Γ the potential u fulfills the Neumann boundary condition

$$\sigma \frac{\partial u}{\partial \nu} = f$$

on Γ. Assuming that Ω is homogeneous except for some inclusions D with constant conductivities smaller than the background conductivity we are concerned with a diffraction problem. Our aim is the reconstruction of D from the knowledge of the Neumann-to-Dirichlet operator Λ_σ. If we denote by Λ the Neumann-to-Dirichlet operator for the domain without inclusions we obtain by the factorization method a characterization of the range of the operator $(\Lambda_\sigma - \Lambda)^{1/2}$. Using this we show that the trace of a function g_z which is essentially the restriction of the fundamental solution of the Laplace equation onto the boundary Γ belongs to $R((\Lambda_\sigma - \Lambda)^{1/2})$ iff z is a point lying in the inclusion. Using the Picard criterion as well as the eigenvalues and eigenfunctions of $\Lambda_\sigma - \Lambda$ this can be tested numerically.

In this presentation we will focus on the numerical implementation of the reconstruction algorithm. As a first step we have to solve the forward problem to simulate the Neumann-to-Dirichlet map. We do this by using boundary integral methods. As the solution for the Laplace equation with given Neumann boundary values f is explicitly known in the half space $\mathbb{R}^3_+$ we only have to solve boundary integral equations on the boundaries ∂D of the inclusions. The boundary integral equations arise from the use of single layer potentials with a modified kernel function on the boundary of the inclusions. For the numerical solution of the integral equations we adapt the spectral boundary integral collocation method proposed by Ganesh, Graham and Sivaloganathan in 1998. Solving the integral equation for several boundary values f gives us an approximation of $\Lambda_\sigma - \Lambda$. In a second step we use this simulated operator to test our reconstruction algorithm. Some results will be given in the presentation.