

A mixed–FEM and BEM coupling for a three–dimensional eddy current problem

S. Meddahi*

V. Selgas*

We study the electromagnetic field generated in a linear and isotropic conductor by a given alternating current density. For a divergence free source current featuring a sinusoidal dependence on time with power frequency, the electromagnetic field can be approximated by a quasi–stationary model. Among these models, the most commonly used is the eddy current model, that can be deduced from Maxwell’s equations neglecting the displacement currents in the Ampère’s law.

Our strategy to estimate the eddy currents is a combination of finite elements (FEM) and boundary elements (BEM). To start with, we reformulate the eddy current problem expressing the electric field in terms of the magnetic field. Then, we split \mathbf{R}^3 into two subregions: the bounded domain Ω_c occupied by the conductor; and the associated unbounded outer region $\Omega_e := \mathbf{R}^3 \setminus \overline{\Omega}_c$. Next, we introduce a scalar magnetic potential in Ω_e that is harmonic and, consequently, has an integral representation. In its turn, this representation provides boundary integral identities relating the trace and the normal derivative of the magnetic potential. Introducing this normal derivative as an auxiliary unknown, one can use the integral identities as non–local boundary conditions for a finite–element treatment in the bounded domain Ω_c . In this region, where the eddy currents are to be computed, we solve directly for the magnetic field and the boundary unknown using edge finite elements and piecewise constant boundary elements, respectively.

We would like to remark that we use a symmetric method for the coupling of BEM and FEM in order to preserve the coercivity of the original problem and, by the way, relax the regularity requirements on the coupling boundary. Indeed, we deduce convergence and optimal error estimates for a Lipschitz domain Ω_c with no restrictions on its topology; cf. [2].

However, when Ω_c is non–simply connected, the implementation of our method requires the resolution of a certain number of auxiliary problems that uniquely depend on the geometry and the topology of the conductor. More precisely, it involves several types of approximations of the harmonic Neumann vector fields. Nevertheless, this difficulty can be overcome introducing a simply–connected computational domain Ω that contains Ω_c and using a Lagrange multiplier associated to the electric field in the dielectric region $\Omega \setminus \overline{\Omega}_c$. This technique is developed in [1] for a mixed–FEM method, whereas its combination with our BEM–FEM coupling is straightforward.

References

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* Departamento de Matemáticas, Universidad de Oviedo. Calvo Sotelo s/n, 33007 Oviedo, Spain;
e–mail: salim@orion.ciencias.uniovi.es selgas@orion.ciencias.uniovi.es

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