A spectral method for integral formulations of potential and high-frequency scattering problems

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The talk will present a fast method for the approximate computation of layer potentials of the form

$$(Kg)(x) := \int_S \frac{\partial^{k+l}}{\partial n_x^k \partial n_y^l} G(x-y) g(y)\, dS_y, \qquad x \in S$$

where $k, l \in \{0, 1\}$.

If the kernel G is smooth and periodic we replace it by the N-term Fourier series expansion. Then fast Fourier transforms can be used to evaluate the potential efficiently.

If G is the Green's function of an elliptic PDE, the Fourier series converges slowly and poor approximations of the potential must be expected. To remedy this situation choose a small parameter $\delta > 0$ and split the Fourier transform of the Green's function into two pieces

$$\widehat{G}(\xi) = \exp(-\delta\,|\xi|^2)\widehat{G}(\xi) + (1 - \exp(-\delta\,|\xi|^2))\widehat{G}(\xi) := \widehat{G}_\delta^S(\xi) + \widehat{G}_\delta^L(\xi)\,.$$

The first part, $G_\delta^S(r)$ is smooth and can be, after multiplication with a suitable cut-off function, approximated by a truncated Fourier series with super-algebraic convergence. The potential due to the second part can be expanded in δ. It can be shown that

$$\int_S G_\delta^L(x-y) g(y)\, dS_y = \sqrt{\delta} g(x) + O(\delta^{3/2})$$

The talk will discuss choices of the parameters δ and N that will lead to asymptotically optimal schemes and present numerical results obtained with BEM formulations of the Laplace and high frequency Helmholtz equation.