

# Efficient calculation of internal results for 2D and 3D elasticity BEM – theory and application

by

Dr.-Ing. Ulrike Eberwien, Prof. Dipl.-Ing. Dr. techn. Gernot Beer

Institute for structural analysis

Graz University of Technology

Austria

The solution of the boundary integral equation in its discretized form requires an accurate treatment of regular as well as weakly and hyper singular integrals. The regular integrals are usually solved numerically using Gauss quadrature. As these integrations make up the majority of the numerical work the appropriate Gauss order chosen by an integration criterion is essential to an accurate and efficient boundary element analysis.

First, the presentation will give an overview on the existing integration criteria, see [1],[2]. By means of a slightly curved boundary element and some internal points  $P_a$  located in the very vicinity of the element (fig. 1(a)) the lack of accuracy concerning the integration of the kernel functions, see eq. 1, will be shown (tab. 1(b)).

$$\Delta U_n^e(P_a) = \int_{-1}^1 \mathbf{U}(P_a, \xi) N_n(\xi) |J(\xi)| d\xi \quad , \quad \Delta \mathbf{T}_n^e(P_a) = \int_{-1}^1 \mathbf{T}(P_a, \xi) N_n(\xi) |J(\xi)| d\xi \quad (1)$$

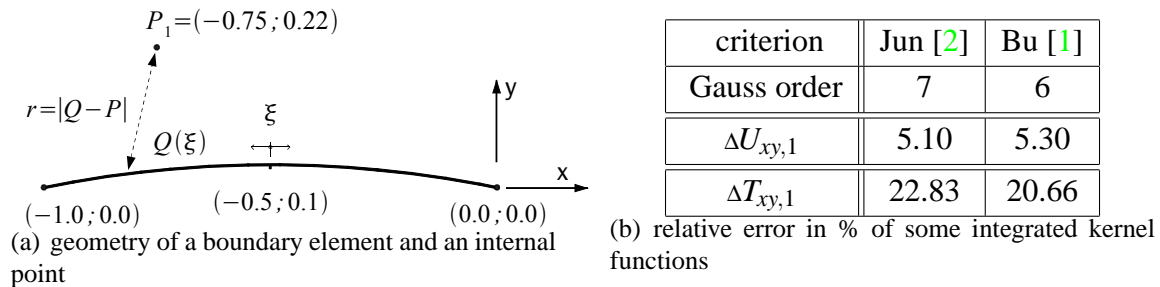


Figure 1: example element and relative errors

As can be seen easily, the error of the integrated kernel functions is rather high. The presentation will reveal a new integration criterion, which allows for a more accurate treatment of the integrals occurring during the calculation of displacements and stresses inside 2D and 3D domains.

In the second part of the presentation the new criterion will be applied to some 2D and 3D benchmark problems.

## References

- [1] S. Bu and T. G. Davies. Effective evaluation of non-singular integrals in 3D BEM. *Advances in Engineering Software*, 23:121–128, 1995.
- [2] L. Jun, G. Beer, and J. L. Meek. Efficient evaluation of integrals of order  $1/r$ ,  $1/r^2$ ,  $1/r^3$  using gauss quadrature. *Engineering Analysis with Boundary Elements*, 2, 1985.