# Using Linear Programming to support High Level Defence Procurement Decisions

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## Abstract

Defence planning within the UK is required to strike a fine balance between aspirations and financial constraints. Delivering defence capability is a complex issue with individual capabilities being intrinsically linked. There is a need to provide advice, at the highest level, to senior Ministry of Defence (MoD) decision makers in adjusting the balance of funding between capability areas to reflect the requirements of an evolving and dynamic defence environment. One method of providing this advice is through Balance of Investment (BoI) Studies which utilise a linear programming approach.

Dstl is currently conducting a BoI, investigating the cost effective balance of Force Elements and enablers required to undertake the full range of required operations, as defined in MoD strategic guidance. Dstl utilises multi-objective optimisation to support this BoI work.

This work is relevant to the conference as it highlights the use of linear programming, on a scale not previously seen in support of Defence in the UK, to inform difficult and often controversial decisions on future equipment procurement options, years in advance of taking delivery.

## 1. Introduction

Demands: What do we need to do?

In recent years the UK Armed Forces have been involved in a wide range of activities, from conflicts in Iraq and Afghanistan, to support for flood victims of the West Midlands in 2007 and the prevention of drug trafficking across the seas. Therefore the aspiration is for the UK Armed Forces to be prepared to undertake a wide range of roles, from high intensity warfare, to peace keeping, to disaster relief. This may be required at both short notice and for long periods of continual activity.

Resources: What do we need to do it with?

Delivering a defence capability is a complex issue with individual capabilities being intrinsically linked. It relies on well trained, organised groups of Soldiers, Sailors and Airmen equipped with a wide range of different military systems, from aircraft carriers, to ambulances to mortars. When on operations service personnel's lives depend on having received the right training and being provided with the right kit at the right place and time.

Funds: Money?

The MoD has a budget of around £34 Billion per year. This is tax payers' money and the UK Armed Forces need to spend the money wisely, ensuring that the equipment being procured is value for money and that the most cost effective choices are made in all areas.

Putting these requirements together, the UK MoD has a significant challenge: it has a set of demands for the types of activities needed to undertake; options for resources; and a monetary budget.

What has been defined is a Balance of Investment problem, the way in which resources are allocated to competing demands to most cost-effectively achieve one or more desired capabilities.

## 2. The Strategic Balance of Investment Study

The Strategic Balance of Investment Study is one of a range of studies designed to provide comprehensive advice at the highest level to Force and Capability Planners in adjusting the balance of funding between capability areas to reflect the requirements of an evolving and dynamic defence environment.

This paper describes the undertaking of this BoI approach to investigate the most cost-effective balance of resources required to undertake the full range of defence demands in accordance with military doctrine.

- Demands and Concepts: The UK defence policy lays out a range of scenarios and tasks required to be undertaken by the UK Armed Forces. Doctrine provides the guiding principles under which the military needs to operate.
- Resources: At the basic level, ships, planes and vehicles are ineffective without trained people to operate and equip them, thus defence uses the term Force Elements to represent the collection of trained people and equipment that can carry out a role. A Force Element may be a trained Company of Infantry man with their weapons and vehicles, or a Navy Frigate with its complement of trained personnel and weapon and sensor suites.

## 3. Method Overview

UK policy provides a set of endorsed planning scenarios to be used in developing the future UK Military Force. A representative subset of the range of operation types, physical environments, threats and coalition partners, that might be expected to arise within current planning assumptions, is used. This is enhanced by a representation of the current operations the UK is undertaking in the near term.

For each scenario, successful campaigns, informed by agreed concept of operation, are generated using high level campaign simulations and wargames. These include the identification of the desired strategic effects and objectives required in each campaign. By phase, the campaigns are then decomposed to identify the required effects and the actions that achieved those effects. Each action is characterised as a task requiring appropriate Force Elements and force enablers to accomplish it. Enablers could include entities such as Engineering Support for Armoured Forces to manoeuvre over obstacles and ISTAR[1].

The Force Elements are then mapped onto the campaign task lists to establish which Force Elements would be capable of undertaking each task. For any given task there are generally a number of different ways of achieving military success using different combinations and levels of Force Elements. For example, the destruction of a building, depending on the range and collateral damage limitations, can be undertaken by artillery, fast jets, UCAV[2] or naval fires. For each task the following process is undertaken;

- The contribution of each potential Force Element that could contribute towards achieving task success is identified;
- An assessment of any additional enabling tasks required to allow a particular Force Element to perform the task, e.g. a mobility task or an ISTAR task, is undertaken;
- The potential Force Elements that could contribute towards achieving the enabling task are assessed.

[1] ISTAR = Intelligence, Surveillance, Target Acquisition and Reconnaissance
[2] UCAV = Unmanned Combat Air Vehicle

The force levels of those elements required, is analysed by drawing on a wide range of more detailed analysis, supplemented by subject matter expert and military judgement where necessary. This process, described schematically in Figure 1, identifies all Force Elements and where there are potential trade-offs for either effectors or enablers.

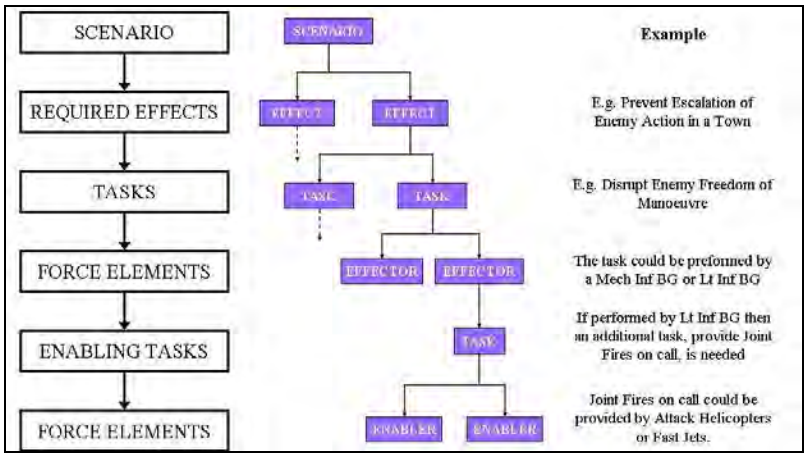


Figure 1: Identification of campaign tasks, effectors and enablers

A number of layers are then added to this capability level:

- General Force Enablers. The associated requirement for general force enablers is also assessed. General requirements cover such areas as strategic transport and Command and Control associated with the deployment of a given force to a military scenario but mainly independent of the tasks the Force Elements will be undertaking.
- Timeframes. A number of timeframes are considered, typically about 5 years in length as demand and resources change over time. In- and out-of-service dates for equipment are defined and the mapping of Force Elements to tasks is repeated for each timeframe as equipment are upgraded and threats evolve.
- Concurrency of Demands. UK Policy requires the military to have the capability to undertake a number of operations simultaneously. Various combinations of scale and type of such concurrent operations are defined, which can vary between timeframes. These concurrency requirements add an additional level of demand above the individual planning scenarios.

A Mixed Integer Linear Programme (LP) has been developed specifically for this study, more details of which are given in Section 4. The LP is used to select the most cost-effective set of Force Elements, known as the force pool. This force pool is capable of generating the force packages to achieve the required effects in each of the scenarios, under various constraints imposed. This can also be thought of as the least cost compliant solution. The LP is able to simultaneously consider the capability requirements across concurrent scenario sets and timeframes. All results are reviewed to ensure that they are militarily coherent. A wide range of sensitivities are preformed on particular areas of equipment performance or cost data, changes to policy or specific task requirements. These sensitivities provide an understanding of the cost-effective force pool being given by the LP, its drivers, sensitivities, key assumptions and possible pivot points. An overview of this method is illustrated in **Error! Reference source not found.**.

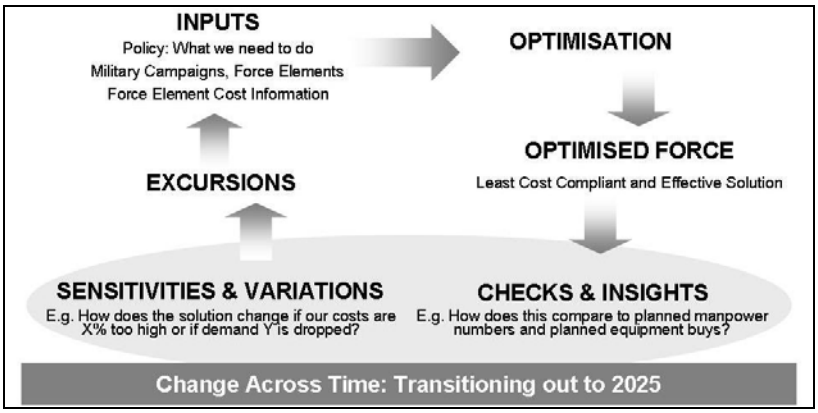


Figure 2: Overview of the Strategic Balance of Investment Study Process

## 4. The Linear Program

At the heart of the study is a bespoke Mixed Integer Linear Program (LP) written in Xpress-MP to assist in the analysis process involving optimisation. The complexity of the model and its reliance on data are such as to allow a rapid investigation of issues of interest to the study. Use of an LP has several benefits for the study, provided a number of assumptions regarding linearity do hold, as shown in Tables 1 and 2.

| Benefits of an LP Approach |
|---|
| Provides a range of options for the most cost-effective force that can achieve all requirements. |
| Provides a clear and auditable relationship between policy, capabilities and the Force Pool. |
| Having all areas in one tool allows for rapid exploration of the solution space which can identify force driving areas and test the sensitivity of the solution to uncertainties in data. |
| All areas can be considered simultaneously, allowing the assessment of complex interactions between multiple capability areas and Force Elements. |

Table 1: Benefits of LP approach

| Assumption | Validity |
|---|---|
| The contributions to overall effectiveness of different force elements are independent and cumulative. | These assumptions are unlikely to be true globally (e.g. at some point adding more aircraft will not increase the effectiveness of the force mix), but are likely to hold on a sufficient subset of the overall problem space. |
| The effectiveness of each force element is linear in the number of the force element used. | |
| The contributions to overall cost of different force elements are independent and cumulative. | Can be arranged to be true by specifying the cost of the force mix in terms of the different force elements. |
| The cost of each force element is linear in the number of the force elements purchased. | Due to, for example, one-off costs and economies of scale, a completely linear form is unlikely to hold. However it has been found that the cost of each force element is well represented by a fixed cost, representing one-off costs for the procurement of the force element, plus a variable cost, representing the unit cost for each Force Element purchased. This gives a piecewise-linear cost function. |

Table 2: Assumptions for linearity and their validity

### The Objective Function

The objective function represents the total cost of the force mix. This cost is minimised, ensuring all tasks can be completed. When constraints in the formulation indicate that there is no feasible way of meeting the full task requirement (e.g. the number of a Force Element that

can be purchased may be limited), a shortfall variable is used to enable a feasible solution to be generated. The objective function takes the form:

$$TOTALCOST = \sum_{f} \left[ \left( X_f \times VarCost_f \right) + \left( B_f \times FixedCost_f \right) \right].$$

The following notation is used:

| | |
|---|---|
| $f$ | An index representing the set of Force Elements |
| $X_f$ | The number of Force Element of type $f$ purchased |
| $B_f$ | A binary variable which is 1 if Force Element $f$ is purchased and 0 otherwise |
| $VarCost_f$ | The variable cost of Force Element $f$ |
| $FixedCost_f$ | The fixed cost of Force Element $f$ |

## LP Capabilities

The LP is flexible and data-driven, and has the following key capabilities:

- No restriction on the type of tasks and Force Elements. This allows the LP to provide insights across mission and domain boundaries, and assess linkages across the Force Structure.
- Representation of enabler tasks, as described in Section 3.
- Representation of the ability of Force Elements to conduct multiple tasks, either concurrently or serially.
- User-specified integerisation. For example, the number of ships deployed may be specified to be an integer, as it is not possible to deploy half a ship, whereas the number of infantry companies deployed do not need to, as fractional deployments are possible.
- Representation of multiple timeframes, which can have different requirements in terms of demand and resource. The optimisation can be conducted at a single timeframe or over multiple timeframes, allowing an assessment of the impact of timing of procurements.

## LP Formulation

The LP represents enabler tasks required to support Force Elements conducting the initial task set. This is achieved by means of a two-stage process, illustrated in Figure 3.

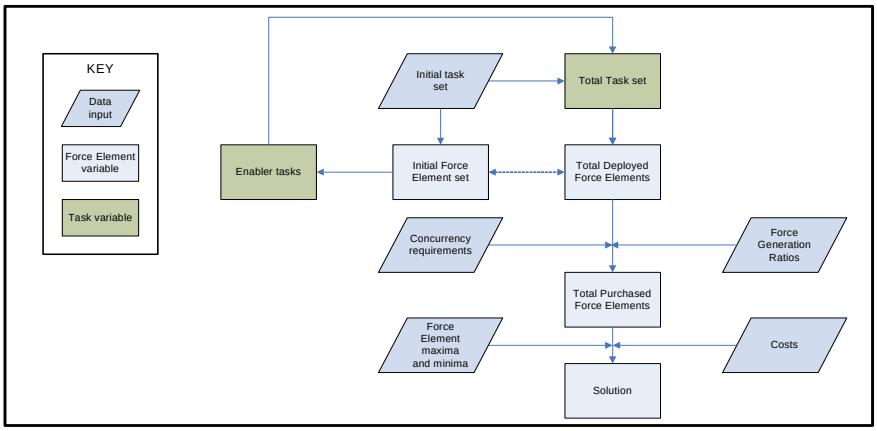


Figure 3: A simplified LP schematic

Stage 1: A set of Force Elements that can complete the initial task set is generated, and the enabler tasks required to support these force elements are calculated. The initial and enabler tasks are then combined to form a total task set.

Stage 2: A second set of Force Elements is generated to complete this total task set. To ensure consistency, the second Force Element set must be a superset of the initial set, and in particular, the Force Elements used for the initial tasks must match in the two sets.

The total purchase requirement is then calculated, taking into account:

- Force generation ratios for each Force Element, i.e. the ratio between the purchased and deployed number. This is used to represent the fact that, for example, a fast jet fleet includes a number of aircraft used for training, or the fact that an infantry company requires recuperation time after being deployed on operations.
- Concurrency requirements, i.e. how many and what types of operation is the force required to conduct concurrently.
- Force Element maxima and minima, used to represent existing fleets or committed purchases across timeframes.

## 5. Outputs

This study provides input into debates on:

- Shortfalls (and affluences) in capability and options for filling those gaps;
- The balance between capability areas;
- Trade-off between Force Elements that support or provide similar capabilities;
- High level cost and performance parameters for major future systems;
- Implications of procurement both numbers and timing decisions;
- Implications for Force Elements and Capabilities from changes in Defence Policy.

Examples of the type of output produced are illustrated in Figure 4.

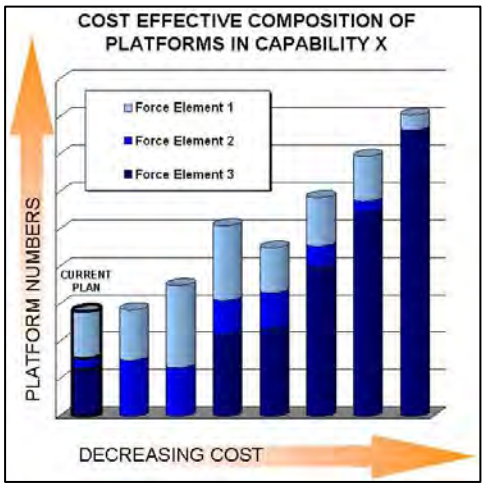


Figure 4: Example Outputs

## 6. Summary

The Strategic Balance of Investment study utilises a bespoke Mixed Integer Linear Programme on a scale not previously seen in support of Defence in the UK. The LP considers millions of variables and constraints and can represent all task types and Force Elements across mission and domain boundaries.

The study is currently providing analysis and insights to senior MoD decision makers to assist in the adjustment of the balance of funding between capability areas, and to support major decisions on equipment procurement options.