Modelling low grazing angle electromagnetic scattering from the sea surface

By R. J. A. Tough and K. D. Ward

Igence Ltd., Malvern, UK

Abstract

We review the integral equation (IE) description of low grazing angle scattering of microwaves, in the main by extended corrugated structures modelling the sea surface and accounting for their effectively infinite extent through the adjunct plane (AP) construction. Solution of the discretised IE by an iterative scheme that incorporates an improved method for the evaluation of the AP contributions is discussed. Perfectly and imperfectly conducting scatterers are considered separately; we attempt to recover the simplicity and physical transparency apparent in the analysis of the former and, to a certain extent, mislaid in earlier discussions of the latter. Results are presented that highlight the impact of shadowing and multiple scattering processes in the low grazing angle scattering regime. Our summary of the underlying mathematical principles of these calculations is complemented by a discussion of several practically useful insights they can provide.

1. Introduction

The modelling of microwave scattering from the sea surface contributes significantly to our understanding and exploitation of maritime radar measurements. Much of this work is based on physically motivated approximations, such as the physical optics, small height perturbation theory and the composite models; in the past fifteen to twenty years progress has also been made in the direct numerical solution of Maxwell's equations, supplemented by appropriate boundary conditions and constitutive relations. The mono-static, low grazing angle geometry typical of surveillance scenarios is of special operational interest; effects of shadowing and multiple scattering are particularly marked in this regime and present the modeller with several interesting challenges. In this paper we discuss how the surface field integral equations for a realistic corrugated sea-surface profile illuminated by a plane wave can be solved to generate useful results reasonably quickly. In this so-called forward/backward (F/B) method [Holliday et al] an iterative solution captures the multiple scattering and shadowing processes. Edge effects induced by the necessarily finite extent of a profile that can be accommodated on a computer are eliminated by the introduction of semi-infinite adjunct planes whose contributions can be specified analytically. The F/B method is rapidly convergent and easily interpreted in physical terms when the sea surface is taken to be a perfect conductor (PC). However the introduction of a large but finite conductivity complicates matters to such an extent that the adjunct plane construction has generally been eschewed in favour of an approach in which finite scatterer size effects are eliminated by introducing a non-uniform weighted illuminating beam [Johnson].

We propose an alternative method that allows us to evaluate both forward and back scattered adjunct plane contributions without recourse to the different and uncontrolled approximations made in [Holliday et al]. It is also more readily adapted to the extra terms that arise when the perfect conductor approximation is relaxed. This provides us with an improved implementation the PCF/B whose extension to the imperfect conductor regime is more straightforward, and preserves the underlying multiple scattering structure of the calculation. Once we have established our methodology we will illustrate its application with an example.

2. The integral formulation of EM scattering

Maxwell's equations, together with appropriate boundary conditions and constitutive relations, describe the propagation and scattering of electromagnetic radiation. Following [Stratton] we can recast these as surface vector field integral equations; in the special case of scattering from a corrugated surface of an incident field whose wave-vector is normal to the grooves in this structure (whose direction we take to define the *y* axis) these reduce to simpler scalar equations in ψ and ζ, the normal component of its gradient

$$\psi(\mathbf{x}) = 2\psi_{in}(\mathbf{x}) + 2\int \left(G_0^{(1)}(\mathbf{x},\mathbf{x}')\zeta(\mathbf{x}') - \psi(\mathbf{x}')\mathbf{n}(\mathbf{x}')\cdot\nabla' G_0^{(1)}(\mathbf{x},\mathbf{x}')\right) dS' \tag{1}$$

$$\psi(\mathbf{x}) = -2\int \left(\alpha\, G_0^{(2)}(\mathbf{x},\mathbf{x}')\zeta(\mathbf{x}') - \psi(\mathbf{x}')\mathbf{n}(\mathbf{x}')\cdot\nabla' G_0^{(2)}(\mathbf{x},\mathbf{x}')\right) dS' \tag{2}$$

Here we identify the quantity α as unity and ψ as the *y* component of the electric field in the Hpol case and the relative permittivity ε of medium 2 and the *y* component of the electric field respectively for Vpol scattering – there is no cross-polar scattering in this specialised geometry. $G_0^{(1,2)}(\mathbf{r},\mathbf{r}')$ are the free space Green's functions describing propagation in the media 1 (above the scattering surface) and 2 (within the scatterer). In the specialised geometry in which the Stratton Chu equations reduce to (1,2)

$$G(\mathbf{r},\mathbf{r}') = -\frac{i}{4} H_0\left(k|\mathbf{r}-\mathbf{r}'|\right) \tag{3}$$

where $H_0(z)$ is a Hankel function. The reciprocity of the Green's function is made quite explicit here. In medium 2 k will take a complex value characteristic of evansescent propagation in an imperfect conductor. The extended boundary condition (2) connects the surface fields ψ and ζ. An approximate evaluation of (2), exploiting the rapid decay in $G_0^{(2)}$ for an imperfectly conducting medium, gives us an impedance boundary condition which decouples (1,2); in the PC limit ψ (Hpol) or ζ (Vpol) vanishes on the scattering surface and the decoupling is immediate and exact. A discussion of the numerical solution of the resulting single integral equation forms the subject of this paper. Once this equation is solved the scattered field at **r** can be evaluated as

$$\psi(\mathbf{r}) = \int \left(G_0^{(1)}(\mathbf{r},\mathbf{x}')\zeta(\mathbf{x}') - \psi(\mathbf{x}')\mathbf{n}(\mathbf{x}')\cdot\nabla' G_0^{(1)}(\mathbf{r},\mathbf{x}')\right) dS' \tag{4}$$

and provides us with the raw material for the analysis of the properties of the sea clutter RCS.

3. The Forward/Back solution for a Perfect Conductor

In the perfectly conducting limit the surface field integral equations (SFIE) reduce to

$$\psi(\mathbf{x}) = 2\psi_{in}(\mathbf{x}) - 2\int \psi(\mathbf{x}')\mathbf{n}(\mathbf{x}')\cdot\nabla' G_0^{(1)}(\mathbf{x},\mathbf{x}')\, dS' \tag{5}$$

for V polarisation; the Hpol equation (expressed in terms of ζ and obtained by differentiating (1) and setting ψ to zero) has a very similar structure. The numerical implementation of the integration implicit in (5), typically though the trapezium rule, converts the integral equation into a set of linear equations that can be represented in matrix form as

$$\mathbf{f} = \mathbf{f}_0 + \mathbf{M}\cdot\mathbf{f} \tag{6}$$

Thus the solution of the SFIE is reduced to the inversion of the matrix **M**. When the scatterer is of finite spatial extent this matrix will itself be of finite size, and standard techniques of numerical linear algebra can be brought to bear on the problem. In our case, where the scatterer is the sea surface, and is illuminated at low grazing angles, significant problems arise in the application of this method. An effectively infinite scattering object cannot be accommodated in a computer of finite capacity; something must be done to reduce the sample of scattering surface to a manageable size. If one were merely to 'chop out' a finite portion of surface, and calculate its RCS, artefacts corresponding to scattering from the resulting sharp edges would dominate the results, particularly in cases where the back-scattering from the surface is relatively weak. Perhaps the most successful approach to this problem has been that of [Holliday et al], which describes the scattering by a finite rough surface, supplemented by adjunct planes that extend to infinity. The contributions of these supplementary surfaces can be calculated, assuming that they support the surface fields established on an infinite planar interface; they vanish identically for points within the sample that have the same elevation as the adjunct planes. These are then incorporated in the driving term $\mathbf{f}_0$; the matrix **M** now need only describe the interactions of the finite portion of ocean surface with the incident radiation and that scattered from the adjunct planes. **M** is constructed from the explicit form of the Green's function (3). [Holliday et al] generate the supplementary driving terms derived from the adjunct planes in much the same way; as we shall see shortly an alternative method based on a spectral representation of $G_0^{(1)}(\mathbf{r},\mathbf{r}')$ is more convenient and readily adapted to the imperfectly conducting case.

The terms above and below the diagonal elements of **M** (the triangular matrices **U** and **L**) respectively describe forward and backward scattering processes; diagonal element contributions can be included in the driving term. This recasts (6) as

$$\mathbf{f} = \hat{\mathbf{f}}_0 + (\mathbf{U}+\mathbf{L})\cdot\mathbf{f} \tag{7}$$

When backscattering is relatively weak solution by the Gauss Seidel method is appropriate, and yields

$$\mathbf{f} = \sum_n \left((\mathbf{1}-\mathbf{U})^{-1}\cdot\mathbf{L}\right)^n \hat{\mathbf{f}}_0 \tag{8}$$

In the case where back and forward scatterings are both significant the matrix identity

$$(\mathbf{1}-\mathbf{U}-\mathbf{L})^{-1} = (\mathbf{1}-\mathbf{L})^{-1}\cdot(\mathbf{1}-\mathbf{U})^{-1} + (\mathbf{1}-\mathbf{L})^{-1}\cdot(\mathbf{1}-\mathbf{U})^{-1}\cdot\mathbf{U}\cdot\mathbf{L}\cdot(\mathbf{1}-\mathbf{U}-\mathbf{L}) \tag{9}$$

suggests an iterative solution of the form

$$\mathbf{f} = \sum_{n=0}^{\infty}\left((\mathbf{1}-\mathbf{L})^{-1}\cdot(\mathbf{1}-\mathbf{U})^{-1}\cdot\mathbf{U}\cdot\mathbf{L}\right)^n\cdot(\mathbf{1}-\mathbf{L})^{-1}\cdot(\mathbf{1}-\mathbf{U})^{-1}\cdot\hat{\mathbf{f}}_0 \tag{10}$$

As the inverses $(\mathbf{1}-\mathbf{L})^{-1},(\mathbf{1}-\mathbf{U})^{-1}$ can be constructed directly by back substitution this solution is as easy to implement as the Gauss Seidel method. The similarity between the Vpol PC SFIE and its Hpol analogue ensures that the former case can be solved in essentially the same way.

4. The Forward/Back solution for an Imperfect Conductor

The impedance boundary condition (IBC) relates the fields ψ, ζ through

$$\psi(\mathbf{x}) \approx \frac{i\alpha\zeta(\mathbf{x})}{k\sqrt{\varepsilon}} \tag{11}$$

so that the the SFIE (1) becomes

$$\psi(\mathbf{x}) = 2\psi_{in}(\mathbf{x}) - 2\int \left(ik\sqrt{\varepsilon}\, G_0^{(1)}(\mathbf{x},\mathbf{x}')/\alpha + \mathbf{n}(\mathbf{x}')\cdot\nabla' G_0^{(1)}(\mathbf{x},\mathbf{x}') \right)\psi(\mathbf{x}')dS \tag{12}$$

We see that this is very similar in form to (5); for Vpol (α=ε) the extra term is a small perturbation. Thus we would expect that the PC F/B approach could be adapted to its solution. In this case the adjunct plane driving term does not vanish for points within the scatterer that are co-planar with the adjuncts. [Holliday et al] expend some effort in eliminating this term; our improved method of APC evaluation is, however, easily generalised to compute it directly. In the Hpol (α=1) case, however, the term introduced by the IBC is large. In the PC case the SFIE satisfied by ζ provided a convenient starting point; the analogous equation obtained when the IBC is invoked is

$$\zeta(\mathbf{x}) = 2\zeta_{in}(\mathbf{x}) + 2\int \left(\mathbf{n}(\mathbf{x})\cdot\nabla G_0^{(1)}(\mathbf{x},\mathbf{x}') - \frac{i}{k\sqrt{\varepsilon}}\mathbf{n}(\mathbf{x})\cdot\nabla\mathbf{n}(\mathbf{x}')\cdot\nabla' G_0^{(1)}(\mathbf{x},\mathbf{x}') \right)\zeta(\mathbf{x}')dS' \tag{13}$$

Here the perturbation term incorporates a rather singular double derivative of the Green's function that contributes to both the **M** matrix and the APCs. Nonetheless a painstaking regularisation [Stoddart] allows us to evaluate the former, while the latter can be computed using the Fourier technique we discuss in the next section. With these problems under control, the extension of the PC F/B method to the imperfect conductor case is relatively straightforward. We note that the computations of [Holliday et al] for both Vpol and Hpol are based on an equation equivalent to (12); this may account for the slow convergence they encounter in the Hpol case. By basing our Hpol computations on (13) we expect to recover much of the rapid convergence of the PC Hpol case.

5. Evaluation of the Adjunct Plane Contributions

When evaluating the APCs [Holliday et al] draw a distinction between those induced by forward and back scattering from the adjunct planes, so that they might better overcome the numerical problems presented by the slowly decaying and oscillatory behaviour of the Hankel functions present in their integrands. The simplest of the APCs, encountered as the contribution form the perturbation introduced by the IBC in the Vpol case, is

$$\Xi_{0,\pm}(\eta,\Delta) = \int_{\Delta}^{\infty} H_0^{(1)}\left(k\sqrt{y'^2+\eta^2}\right)\exp(\pm ik\sin\theta y')dy' \tag{14}$$

They compute the backscattered ('+') contribution using a high frequency approximation derived by dint of a succession of integration by parts, and whose higher order terms rapidly become intractable. The forward scattered ('-') contribution requires a different approach: that part of the integrand that decays most slowly with y' is identified, subtracted off and integrated analytically. The remaining part of the integrand is then integrated numerically and the results are added together. This approach is rather ungainly, if only because it treats the forward and back APCs differently and invokes some rather uncontrolled approximations. An alternative method of evaluation, that can be extended, with a little ingenuity, to all the APCs that we have encountered in our development of the F/B method, is as follows. To this end we introduce a contour integral representation of the Hankel function ,

$$H_0^{(1)}\left(k\sqrt{y'^2+\Delta^2}\right)=\frac{1}{i\pi}\int_{-\infty-i\pi/2}^{\infty+i\pi/2}\exp\left(ik\left(y'\cosh q+i\Delta\sinh(q)\right)\right)dq \tag{15}$$

so that

$$\Xi_{0,\pm}(\eta,\Delta)=\frac{1}{\pi k}\int_{-\infty-i\pi/2}^{\infty+i\pi/2}dq\,\frac{\exp\left(ik\left(\Delta\left(\cosh(q)\pm\sin\theta_{inc}\right)+i\eta\sinh(q)\right)\right)}{\cosh(q)\pm\sin\theta_{inc}} \tag{16}$$

If we parameterise the contour of integration as

$$q=t+i\tan^{-1}(t),\quad dq=dt\left(1+\frac{i}{1+t^2}\right) \tag{17}$$

then

$$\Xi_{0,\pm}(\eta,\Delta)=\frac{1}{\pi k}\int_{-\infty}^{\infty}dt\left(1+\frac{i}{1+t^2}\right)\frac{\sqrt{1+t^2}}{\cosh t+it\sinh t\pm\sqrt{1+t^2}\,\sin\theta_{inc}}\times$$
$$\times\exp\left[\frac{ik}{\sqrt{1+t^2}}\left\{\Delta\left(\cosh t+it\sinh t\pm\sqrt{1+t^2}\,\sin\theta_{inc}\right)+i\eta\left(\sinh t+it\cosh t\right)\right\}\right] \tag{18}$$

The integrand in the real space representation (14) is oscillatory, slowly decaying and problematical; the t space realisation (18) merely requires a numerical quadrature of a localised and well-behaved function. This strategy, with a few simple variations, allows us to evaluate all the APCs we have encountered in this work.

6. Scattering from the sea surface

The sea surface is characterised by height fluctuations on many length scales. These range from small ripples on the scale of millimetres up to ocean swell of hundreds of metres. Using a radar with a wavelength of 3 cm at a low grazing angle (here we use 3°) it is found that most backscattering comes from the small scale roughness, which can be caused by wind ripples or the roughness of breaking waves. A useful technique to investigate the characteristics of the scattering is to simulate the surface without the ripples, and then to calculate the backscatter as ripples are added to different parts of the surface. Figure 1 shows a simulation of 20m of sea surface without ripples, and the radar illumination shown as red rays. Note that there are shadowed areas.

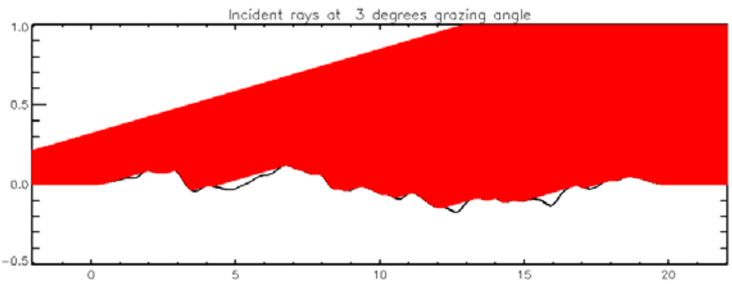

Figure 1. Sea surface simulation with incident radiation represented by rays.

The surface in Figure 1 looks 'smooth' at the radar wavelength and the surface gradient is never sufficiently large to produce specular backscatter. Numerical EM calculation using F/B, including careful computation of the adjuct plane contribution, gives the expected result of negligible backscatter. A small patch of roughness is added to the surface and the backscatter is recalculated. Figure 2 shows the resulting radar cross section (RCS) for vertical (black line) and horizontal (red line) polarisations as a function of the position of the rough patch on the surface. The fluctuations are caused by the local gradient, shadowing and multiple scattering on the smooth surface. These results may be compared with Figure 3, which shows the so-called composite model, where geometric shadowing is assumed and only the local gradient of the smooth surface is accounted for. The differences between Figure 2 and 3 show the deficiencies of the composite model and the benefit of numerical calculations using the techniques outlined above in sections 1 to 5.

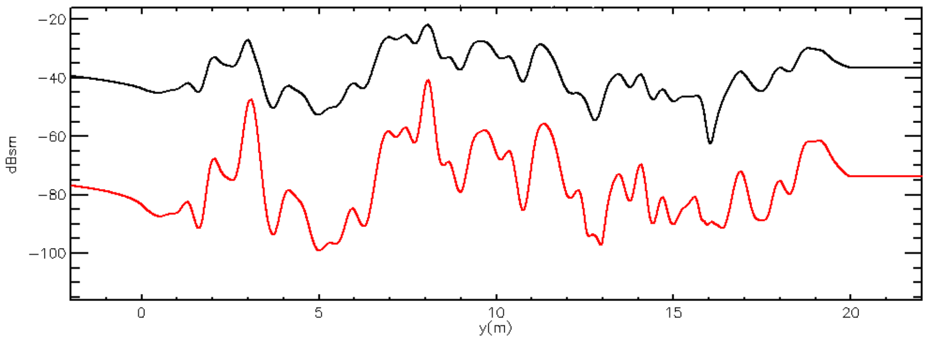

Figure 2. F/B calculation of the RCS as a function of the position of the small scale roughness.

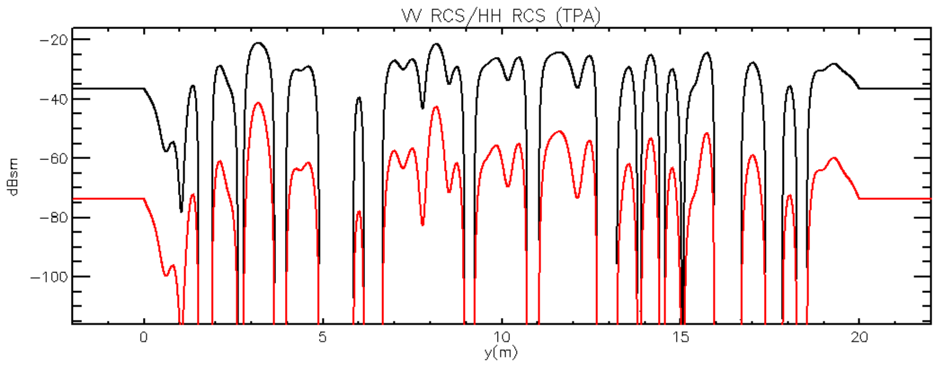

Figure 3. Composite model calculation as a function of the position of the small scale roughness.

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