The MIMO channel Ortogohalizations applying universal eigenbasis

V. Kontorovitch

Electrical Engineering Department, Communications Group, CINVESTAV-IPN.
Av. IPN # 2508, Colonia San Pedro Zacatenco, C.P. 07360, México D.F.
Phone (52) +55 50613764, Fax (52) +55 57477088.
e-mail: valeri@cinvestav.mx

Abstract

This paper presents an overview of the mathematical fundamentals of the MIMO channel orthogonalizations, that use the Prolate Spheroidal Wave Functions (PSWF) as a universal basis. Opposed to the optimum basis, based on the Karhunen-Loéve expansion (KLE), the PSWF basis is invariant to the covariance properties of the MIMO channel matrix and provides an almost minimum set of eigenfunctions that allow to represent the channel with a predefined accuracy.

I. Introduction and MIMO channel model

The MIMO (Multiple Input-Multiple Output) channel deals with many antennas at both sides of the link (transmission and receiving terminals).

The base-band relationship for MIMO broadband channels considered hereafter is (see Yu & Ottesen 2002):

$$\dot{\boldsymbol{z}}(t,\boldsymbol{r}) = \dot{\boldsymbol{H}}(t,\tau,\boldsymbol{r}) \otimes \dot{\boldsymbol{S}}(t,\boldsymbol{r}) + \dot{\boldsymbol{n}}(t,\boldsymbol{r}) \qquad (1.1)$$

where $\otimes$ is a convolutional operator, $\dot{\boldsymbol{H}}(\cdot)$ is an N x M channel impulse response matrix; N, M are the number of transmitting and receiving antennas respectively; $\boldsymbol{z}(\cdot)$ is an aggregated received signal; $\boldsymbol{S}(\cdot)$ is the transmitted signal; $\boldsymbol{n}(\cdot)$ is an additive noise. Here τ and $\boldsymbol{r}$ are delay of the channel and radii-vector of the receiver location. As it is seen from (1.1) the MIMO channel is sufficiently vectorial.

For narrow band case:

$$\dot{\boldsymbol{H}}(\cdot) \cong \dot{\boldsymbol{H}}(t,\boldsymbol{r})\,. \qquad (1.2)$$

Supposing here that the elements $\dot{h}_{i,j}(\cdot)$ of $\dot{\boldsymbol{H}}(\cdot)$ are scalar Gaussian random fields (in general case), the MIMO channel can be characterized by the covariation matrix $\boldsymbol{R}_H$ (see Weichselberger, et al. 2006):

$$\boldsymbol{R}_H = < vec(\dot{\boldsymbol{H}}) vec^H(\dot{\boldsymbol{H}}) >, \qquad (1.3)$$

where $\dot{\boldsymbol{H}}$ has N x N size, < > is an averaging symbol and $(\cdot)^H$ denotes the complex conjugate transpose.
For many cases the so-called Kronecker model can be assumed:

$$\boldsymbol{R}_H = \boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Tx}} \otimes \boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Rx}}\,, \qquad (1.4)$$

where the symbol $\otimes$ denotes the Kronecker product of the covariance matrixes at both ends (transmitter and receiver):

$$\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Tx}} = < (\dot{h}_i \dot{h}_j)^T >;\ j = \overline{1,N} \quad ; \quad \boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Rx}} = < \dot{h}_j \dot{h}_i^H >;\ j = \overline{1,M}\,, \qquad (1.5)$$

where $\dot{h}_i$ is the i-th row of $\dot{\boldsymbol{H}}$, $\dot{h}_j$ is the j-th column of $\dot{\boldsymbol{H}}$ and T is the symbol of the matrix transpose and * - is the complex conjugate symbol.

For many practical cases $\boldsymbol{R}_H$ can be considered as a separable in the space and time domains, though $\boldsymbol{R}_H$ is assumed to be a matrix product of $\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{t}}$ and $\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{s}}$ (t-time, s-space).This case is considered in the sequel. For the Kronecker model $\dot{\boldsymbol{H}}$ can be presented in the following way:

$$\dot{\boldsymbol{H}} = \left(\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Rx}}\right)^{1/2} \boldsymbol{G} \left(\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Tx}}\right)^{H/2} \quad ; \tag{1.6}$$

where $\boldsymbol{G}$ is a diagonal matrix of i.i.d. Gaussian variables.

The generalized Kronecker model is (see for example Weichselberger, et al. 2006; Alcocer, et al. 2006):

$$\dot{\boldsymbol{H}} = \left(\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Rx}}\right)^{1/2} \left[\tilde{\Omega} \bullet \dot{\boldsymbol{H}}(t,\tau)\right] \left(\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Tx}}\right)^{H/2} \quad ; \tag{1.7}$$

where $\dot{\boldsymbol{H}}(t, \tau)$ is a matrix of impulse responses in the time domain, $\tilde{\Omega}$ is the element-wise square root of the so-called "coupling matrix" Ω, which physically represents, how the eigen-modes of propagation (between transmitter and receiver) are "connected" through the scattering environment. For many cases (at least for the channel orthogonalization approach), $\dot{\boldsymbol{H}}(t, \tau)$ can be tackled as diagonal matrix of the statistically independent Gaussian SISO (Simple Input-Simple Output) channels[1]. Note that:

$$\boldsymbol{R}^{1/2}(\boldsymbol{R}^{1/2})^* = \boldsymbol{R}\,.$$

Then $\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{s}}$ is (see Weichselberger, et al. 2006):

$$\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{s}} = \begin{bmatrix} \boldsymbol{R}_{1,1}^{Rx} & \cdots & \boldsymbol{R}_{1,N}^{Rx} \\ \vdots & & \vdots \\ \boldsymbol{R}_{N,1}^{Rx} & \cdots & \boldsymbol{R}_{N,N}^{Rx} \end{bmatrix} \tag{1.8}$$

where:

$$\boldsymbol{R}_{\boldsymbol{m_1},\boldsymbol{m_2}}^{\boldsymbol{Rx}} = \langle (h_{col,m_1}) \cdot (h_{col,m_2}) \rangle\,,$$

m_1 and m_2 are numbers of the receiver antennas; $h_{col,1} = \{h_{i,j}\}$. Then (1.7) can be also represented in the way:

$$\begin{aligned} \boldsymbol{R}^{Rx} &= \langle \dot{\boldsymbol{H}}\dot{\boldsymbol{H}}^{H} \rangle = \sum_{m=1}^{N} \boldsymbol{R}_{m,m}^{Rx} \\ \boldsymbol{R}^{Tx} &= \langle \dot{\boldsymbol{H}}^{T}\dot{\boldsymbol{H}}^{*} \rangle = \sum_{n=1}^{M} \boldsymbol{R}_{n,n}^{Tx} \end{aligned} \tag{1.9}$$

Hereafter, take into account that the link is assumed to be reversible and in consequence it is possible to consider only one of the terminals.

2. Orthogonalization principle

In order to successfully apply the signal processing algorithms for the MIMO channel it is necessary to propose some approaches for the constructive representations of the $\dot{\boldsymbol{H}}(\cdot)$, as in the MIMO case it is sufficiently multidimensional. As Parra et al. (2005) mention it one of the most feasible ways to do this is the so-called channel orthogonalization approach, i.e. formation of the finite set of the space-time "artificial trajectories", which gives a minimum set of entries for $\dot{\boldsymbol{H}}$. In the case when $\boldsymbol{R}_H$ fulfills the separability condition, those artificial trajectories are artificial multipath trajectories in the time domain and artificial eigenmodes for the space propagation in the spatial domain as well. Notice that, it is possible to make orthogonalizations in both the transmitting and receiving terminal, however, hereafter we will consider only the receiving point of the link. It is well known that in the receiver the orthogonalization can be achieved applying the KLE method. The main drawback of this method is also well known: the orthogonalization basis strongly depends on the covariance matrix $\boldsymbol{R}_H$ (or its components $\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{t}}$, $\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{s}}$). The challenge is to avoid this main drawback and preserve the attractive features of the KLE: minimum basis that allows to represent $\dot{\boldsymbol{H}}(\cdot)$ with a predefined mean square error (MSE). The solution was found first for SISO channels by Parra et al. (2005) and then it was extended to the MIMO case by proposing particularly the PSWF basis as a "universal"

[1] See the corresponding material in the sequel

model for both spatial and time domains (see Alcocer, et al. 2005; Alcocer, et al. 2005b). The concept of "universal basis" means that the parameters of the eigen-basis do not depend on the covariance matrix $\boldsymbol{R}_H$, but depend only on the "external" parameters of the communication link such as: the number of antennas, the antenna aperture, the system bandwidth, the symbol interval, etc.

The "universal" basis is not unique of course, but the PSWF satisfy all the requirements as universal basis for the channel ortogonalizations that provides practically a minimum set of eigen functions with a predefined MSE for the "limited" bandpass equivalent MIMO channel models. For the last case, PSWF universal basis offers a fully generic MIMO channel model which can be characterized with a minimum set of statistically independent (in Gaussian case) components (see for example Kontorovich 2006; Alcocer et al. 2006b). The latter provides "full diversity" properties for the model together with the minimum complexity (respect to algorithms with equal conditions) for the signal processing.

3. Main result

Let us consider $\boldsymbol{R}_H^t$,and $\boldsymbol{R}_H^s$ separately. For the $\boldsymbol{R}_H^t$ orthogonalization it can be immediately shown, that in delay and Doppler domains, PSWF basis come out directly. For example, for the SISO channel let us consider the input delay spread function $h(t, \tau)$ and the transfer function $\tilde{H}(j\omega,t)$, that are connected through the Fourier transform $\tilde{h}(t,\tau) \underset{\tau}{\overset{F\{\}}{\Leftrightarrow}} \tilde{H}(j\omega,t)$, where $F\{\}$ represents the operator of the Fourier transform, $\omega = 2\pi f$, $\tau \in [-T/2, T/2]$, $f \in [-T/2, T/2]$. Defining the covariance function for $h(t, \tau)$ in the following way:

$$R_h(t,\tau) = \int_{-F/2}^{F/2} \left|\tilde{H}(j\omega,t)\right| \exp(j\omega\tau)d\omega\,, \tag{3.1}$$

for $|\tilde{H}(j\omega,t)|^2 \cong K_0 = \text{const}$ (flat and slow fading conditions), one can find the KLE integral equation (for details see Van-Trees 2002]):

$$\int_{-T/2}^{T/2} \varphi(t) P \frac{\sin[2\pi F(t-\tau)]}{F(t-\tau)]} dt = \lambda\varphi(t) \tag{3.2}$$

where $P = K_0 F$, $\{\varphi(t)\}$-are eigen functions, $\{\lambda\}$- are eigen-basis. In equation (3.2) the eigen basis are PSWF (see Van-Trees 2002]). Similarly, as Zemen & Mecklenbräuher (2005) point it, the Doppler domain can be represented using PSWF. Note that in the above mentioned cases PSWF is an uncorrelated basis, whose parameters depend only on the external parameters for example F and T. In the case of selective fading it is necessary to consider the asymptotic representation for PSWF (when $F \to \infty$) in the sinc-function form. The last one provides uncorrelated entries for $\tilde{\boldsymbol{H}}$ as well. Then, due to the fact that in time domain PSWF can be considered as universal basis and that SISO channels are statistically independent in (1.7), all the above ideas are valid for MIMO channels as well (the proof that PSWF are practically equivalent to KLE for different power-delay profiles can be found in Parra et al. 2002; Alcocer et al. 2005b).

Now it is time to explain why, the SISO channels, elements of $\tilde{\boldsymbol{H}}(t,\tau)$, are statistically independent. We will analyze the $\boldsymbol{R}_H^s$ orthogonalizations. For simplicity let us consider the uniform linear antennas (ULA's) in both sides of the link. Then it is easy to show (see Alcocer et al. 2005; Alcocer et al. 2005b for details) that the space elements of $\boldsymbol{R}_H^s$ for the receiving terminal ($\boldsymbol{R}_H^{Rx}$) can be represented as follows:

$$b_{l,\rho} = \text{const}\,\text{Re}\left\{ \int_{-\frac{\Delta\theta}{2}}^{\frac{\Delta\theta}{2}} \left|G_R(\theta)\right|^2 W(\theta) \exp(j\,\tfrac{2\pi d}{\lambda}\sin\theta) d(\sin\theta) \right\}, \tag{3.3}$$

where l,ρ antenna elements; $l,\rho=\overline{1,K}$, $W(\theta)$ is the PDF of the angle of arrival (AoA), G_R (θ) is the antenna pattern of the receiving ULA, d is the antenna spacing, λ is the wave length, $\theta \in [\theta_{min}, \theta_{max}]$ and $\Delta\theta = [\theta_{max} - \theta_{min}]$.

One can see that (3.3) is an incomplete Fourier transform of $|G_R(\theta)|^2 W(\theta)$ and their eigenbasis is PSWF. When $\Delta\theta$ is limited, then there is an isolated group of eigenvalues with the effective rank r_{eff}:

$$r_{eff} = E\left\{2\frac{\Delta\theta D_a}{\lambda}\right\}+1, \tag{3.4}$$

where E{} is integer and D_a is the dimensionality of ULA. Actually r_{eff} is the number of PSWF considered for the representation in (3.3).

For (3.3) with (3.4), the space part of $\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Rx}}$ is ([5, 6] and the references therein):

$$\boldsymbol{R}_{\boldsymbol{H}}^{\boldsymbol{Rx}} = \boldsymbol{U}\Lambda\boldsymbol{U}^H , \tag{3.5}$$

where $\boldsymbol{U}=[\boldsymbol{U}_1|...|\boldsymbol{U}_{r_{eff}}]$ is a column matrix (K x r_{eff}), Λ is a diagonal matrix of eigenvalues with rank r_{eff}, K is the number of antenna elements. Then, by means of (3.5) it is possible to create the space part of the model in (1.7) in the **fully generic way** with uncorrelated entries for the SISO part in the time domain. As Weichselberger, et al. (2006) and Alcocer et al. (2006) show it, the coupling matrix Ω can be constructed as:

$$[\Omega]_{m.n} = \left\langle\left\{\left|\varphi_{Tx,m}^H \Theta \varphi_{Rx,n}^*\right|^2\right\}\right\rangle , \tag{3.6}$$

where Θ is a power azimuth spectrum (PAS) and { φ } denotes the PSWF for the spatial domain. So, Ω demonstrates how the artificial eigen-modes { φ } are "connected", i.e. how many statistically independent SISO channels have to be considered for the model of the MIMO channel. The generalization of the above material for any planar antennas (PA) at both extremes of the link is reported by Parra et al. 2002; Alcocer et al. (2005), (2005b), (2006), (2006b); and Kontorovich (2006).

To conclude it is necessary to remark that all theoretical statements made here are completely confirmed by simulations (for details see Parra et al. 2002; Alcocer et al. 2005, 2005b).

4. Conclusions

4.1. The universal basis approach for MIMO channel orthogonalizations in the PWSF form was presented.

4.2. This universal basis forms a fully generic model for the MIMO channel in both time and space domain.

4.3. The fully generic MIMO channel model is an essential point for the creation of space-time processing algorithms for MIMO channel simulation, optimum RAKE-type receivers, space-time coding, etc.

REFERENCES

YU K. & OTTESEN B. 2002. Models for MIMO propagation channel, a review, *Wiley Journal on Wireless Communications and Mobile computing (Special issue on Adaptive Antennas and MIMO Systems).*

WEICHSELBERGER, W., HERDIN M., ET AL. 2006. A stochastic MIMO channel model with joint correlation of both link ends, *IEEE Trans on Wireless Communications* **5 (1) January**, 90-100.

ALCOCER, A., PARRA, R. & KONTOROVICH, V. 2006. A MIMO channel simulator applying universal basis, *VTC-Fall'06, Canada*.

PARRA R., KONTOROVICH V., OROZCO-LUGO A. 2002. Modelling wide band channels using orthogonalizations, IEICE Trans. V. E. –852 (3), 544-551.

ALCOCER, A., PARRA, R. & KONTOROVICH, V. 2005. Some general properties of the covariation matrix for MIMO communication channels, *IST Mobile wireless Communicatioon summit, Germany*.

ALCOCER, A., PARRA, R. & KONTOROVICH, V. 2005b. An orthogonalization approach for communication channel modelling, *VTC'05 (fall), Dallas, USA, September*.

KONTOROVICH V. 2006. Orthogonalization approach to channel modeling, *Int. summer workshop 2006, "Propagation and channel modeling", Norway*.

ALCOCER, A., PARRA, R. & KONTOROVICH, V. 2006b. A fully generic wideband MIMO channel simulator, *Submitted to IEEE Int. conference*.

VAN-TREES, H. 2002. Detection, estimation and modulation theory, *Part IV, Wiley*.

ZEMEN, T. & MECKLENBRÄUHER, C.F. 2005. Time-variant channel estimation using discrete prolate spheroidal sequences, *IEEE Trans. on Signal Processing* **53(9), September**, 3597-3607.