

# Detecting non-stationary signals using fractional Fourier methods

By A. T. Catherall and D. P. Williams

Defence Science and Technology Laboratory, Physical Sciences, Winfrith Technology Centre,  
Dorchester DT2 8WX.

## Abstract

Signal processing methods have been developed over the last 60 years to detect and analyse complicated non-stationary signals, such as speech and seismic activity. The traditional method for analysing such signals is through a spectrogram based on the short-time Fourier transform (STFT). However, the STFT is not ideal since it reflects only the stationary properties contained in any short time-segment of the signal. Other methods, such as wavelet transforms and Wigner-Ville distributions, which have found favour in speech processing, inevitably produce cross-term interference resulting in the appearance of false signals. New processing techniques are required to produce improved spectrograms for analysis of non-stationary signals.

Recently, spectrograms based on the fractional Fourier transform (FrFT) were shown to give better time-frequency resolution of signals containing a non-stationary components, such as a linear chirp. However, current FrFT methods are not always ideally suited to signals containing multiple non-stationary components. In this paper, we present an improved spectrogram based on the FrFT, which accounts for signals with multiple non-stationary components. To demonstrate the superior time-frequency resolution of such signals using our improved spectrogram, we analyse signals containing multiple non-stationary components, including an artificial signal and a bat echolocation signal.

## 1. Introduction

The spectrogram is one of the most useful and widely used tools in signal processing. The spectrogram is, like the very first sound spectrograph, the running output of the Fourier transform of a signal, typically defined as the intensity of the short-time Fourier transform (STFT). The classical means to calculate the STFT (and spectrogram) involves windowing a time-series signal into discrete intervals in time, which are each Fourier transformed in turn to obtain the power spectrum of the signal as a function of time. However, the spectrogram calculated this way is not well suited to the analysis of non-stationary signals. It is well known that long analysis windows cause a moving signal to be “smeared” over several frequency bins resulting in a poor time resolution, whilst a short analysis window cannot always achieve the desired frequency resolution.

Over the last 60 years, numerous attempts have been made by various authors to improve upon the spectrogram, including wavelet analysis, the Wigner-Ville transform and its modifications, and reassignment methods (e.g. Fulop and Fitz 2006). However, these other methods have not been widely adopted by the signal processing community since they either suffer from the appearance of unwanted cross-terms, may be difficult to interpret or are cluttered with meaningless random points – all obstacles in the way of reading the spectrogram.

In recent years, spectrograms based on FrFTs have been identified as methods for improved analysis of non-stationary signals (Capus and Brown 2003; Vargas-Rubio and Santhanam 2005). However, most existing FrFT methods are not ideally suited to signals containing multiple non-stationary components, especially signals with multiple non-related chirps. In

this paper, we present a method for producing an improved spectrogram, which uses an optimal FrFT for each detected component in a signal. Our optimisation method is ideally suited for signals which contain several non-stationary (or moving) components. To demonstrate the superiority of our improved method over the STFT, Wigner-Wille and existing FrFT methods, we analyse an artificial multi-component signal and a real signal from a bat. The results obtained from each processing method are displayed and discussed.

## 2. The Fractional Fourier transform (FrFT)

The fractional Fourier transform (FrFT) of a function  $f(t)$  is given by

$$\mathfrak{F}_\alpha(x) = \frac{e^{-i(\pi/4 - \alpha/2)}}{(2\pi|\sin \alpha|)^{\frac{1}{2}}} \exp\left(\frac{1}{2}ix^2 \cot \alpha\right) \int_{-\infty}^{\infty} \exp\left(-\frac{ixt}{\sin \alpha} + \frac{1}{2}it^2 \cot \alpha\right) f(t) dt \quad \text{eq (1)}$$

where  $\mathfrak{F}_\alpha$  is the fractional transform of order  $\alpha$ ,  $t$  is time and  $x$  is the transform domain. When  $\alpha = \pi/2$ , the transform reduces to the standard Fourier transform and  $x$  represents frequency. The FrFT of a linear chirp is an impulse response when the value of  $\alpha$  is selected to match the chirp rate.

### 2.1 Short-Term Fractional Fourier Transform (STFrFT)

A spectrogram based on a short-time fractional Fourier transform (STFrFT) analysis offers improved resolution of chirp signals over conventional STFT analysis (Capus and Brown 2003). If the signal properties are known in advance, the optimum  $\alpha$  used may be predetermined. Alternatively,  $\alpha$  can be tuned with reference to the entire signal (globally optimised), or be selected independently for each window of analysis (locally optimised). Although tuning  $\alpha$  to its optimum value may be quite computationally intensive, such a method has the advantage of requiring no *a priori* knowledge of the signal (unlike a matched filter). However, the described methods of analysis are still not ideal, since the value of  $\alpha$  used will only be optimal for one signal component, and will be sub-optimal for any signal components moving with different chirp rates. Hence, these methods are not well suited to signals containing multiple non-related chirps.

### 2.2 Component optimised STFrFT (CoSTFrFT)

In the STFrFT approach proposed here, the value of  $\alpha$  is optimised for each signal component, whether stationary or moving. FrFTs are performed over a range of orders  $\alpha$  in a short-time analysis window of the signal. The  $\mathfrak{F}_\alpha(x)$  giving the maximum magnitude response is located, which identifies the strongest component in the signal. The original signal with the identified component removed is reconstructed by setting the identified  $\mathfrak{F}_\alpha(x)$  to zero, and performing the inverse FrFT (Bennett and McLaughlin 2004). This process is iterated until no more signal components above a certain amplitude threshold are detected. The output is then constructed based on each of the identified  $x$  and  $\alpha$  values. The whole process is repeated for each short-time analysis window to generate a new time-frequency representation analogous to a spectrogram.

### 3. Signal analysis

For illustration purposes, we apply our processing method to two signal types and compare its performance against other processing methods including the STFT, Wigner-Ville (W-V), pseudo Wigner-Ville (PW-V), smoothed pseudo Wigner-Ville (SPW-V), and a locally optimised STFrFT (see 2.1).

#### 3.1 Multi-component chirp signal

The first example has been constructed to test the resolving ability of each method; the signal consists of a linear and a quadratic chirp. The time-frequency representation of the signal is shown in Figure 1. The STFT is clearly non-ideal for displaying the content of this signal: both chirps are highly unresolved and cannot be separated when close in frequency. The W-V and PW-V methods, although offering some improved resolution, suffer from severe cross-term interference (Figures 1(b) and 1(c)). Cross-term interference effects are substantially reduced using the SPW-V scheme, however the resulting spectrogram is only marginally more resolved than that obtained using the STFT, Figure 1(d). The locally optimised STFrFT offers sufficient resolution to identify that the two chirps do not cross over in frequency. However, the spectrogram is only optimised for the strongest identified component in the signal, which is the linear chirp in this case. The quadratic component therefore appears smeared when its chirp rate differs from that of the linear chirp. The CoSTFrFT resolves both the quadratic and linear chirps throughout their duration, providing a more resolved and informative spectrogram. For example, it is clear that the quadratic component is slightly weaker, throughout its entire duration, than the linear component.

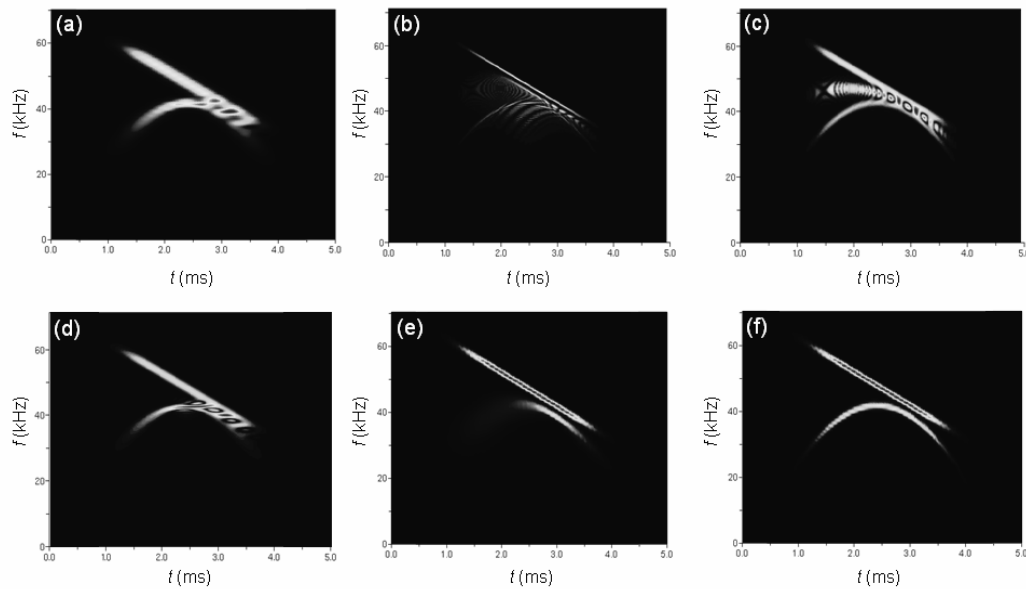


Figure 1: Time-frequency representation of the multi-component chirp signal: STFT (a); W-V (b); SW-V (c); PSW-V(d); Locally optimised STFrFT (e); Component Optimised STFrFT (f).

### 3.2 Beckman bat signal

We now apply the processing schemes to a recording of a bat signal<sup>1</sup>. This signal contains several approximately harmonically spaced and almost linear chirps. Figure 2 shows the time-frequency representation of the signal using the various methods. The STFT spectrogram again offers poor time-frequency resolution, and the W-V and PW-V methods suffer from cross-term interference problems. The problems of optimisation are quite apparent for the locally optimised STFrFT. Clearly this method offers better resolution than the standard STFT, however a discontinuity is apparent at  $t = 2.0$  ms; this is a result of the optimisation being based on the strongest component, which changes during the duration of the signal. The CoSTFrFT suffers from no such problem, producing a clearer spectrogram.

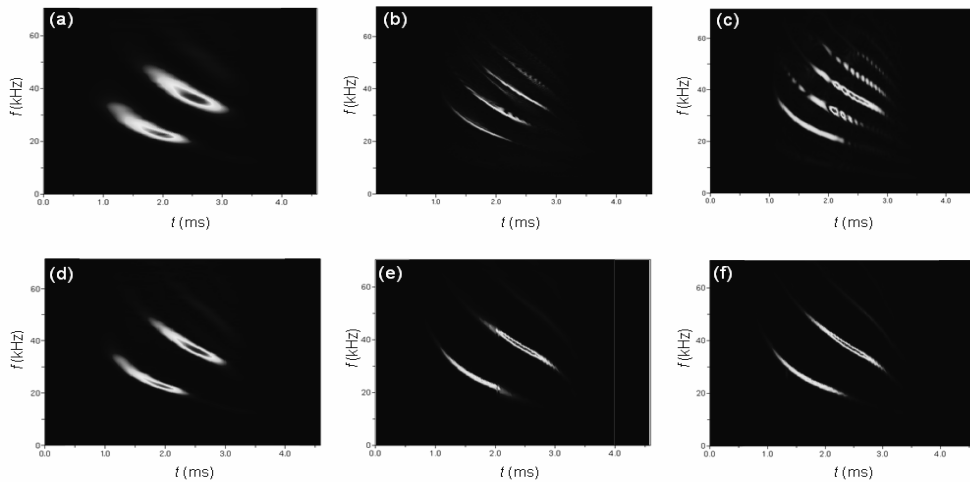


Figure 2: Time-frequency representations of the Beckman bat signal: STFT (a); W-V (b); SW-V (c); PSW-V(d); locally optimised STFrFT (e); Component Optimised STFrFT (f).

## 4. Conclusion

A method for processing signals based on optimising the fractional order for each signal component has been presented. We have demonstrated that our processing scheme offers spectrograms with improved resolution of signals containing multiple non-linear chirps in comparison to several other established methods. However, the improvements do come at a cost: the CoSTFrFT is more than an order of magnitude more time consuming than the STFT.

## REFERENCES

- BENNET, M. & McLAUGHLIN S. 2004. Filtering of chirped ultrasound echo signals with the fractional Fourier transform. *IEEE International Ultrasonics, Ferroelectrics and Frequency control joint 50<sup>th</sup> anniversary conference*. 2036-2040.
- CAPUS, C. & BROWN K. 2003. Short-time fractional Fourier methods for the time-frequency representation of chirp signals. *J. Acoust. Soc. Am.* **113** (6), 3253-3263.
- FULOP, S. A. & FITZ, K. 2006. Algorithms for computing the time-corrected instantaneous frequency (reassigned) spectrogram, with applications. *J. Acoust. Soc. Am.* **119** (1), 360-371.
- VARGAS-RUBIO, J. G. & SANTHANAM B. 2005. An improved spectrogram using the multiangle centered discrete fractional Fourier transform. *IEEE Proceedings ICASSP '05* **4**, 509-512.

<sup>1</sup> We wish to thank Albert Feng of the Beckman Institute at the University of Chicago, Illinois, for permission to use the bat data.