Modelling different failure modes in CBM applications using a weighted combination of stochastic filters

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Abstract: In the context of condition-based maintenance (CBM), probabilistic stochastic filters provide an established means of recursively estimating the residual life of an individual component using condition monitoring (CM) information. In this paper, we consider the potential for modelling the impact of multiple failure modes that exhibit specific types of behaviour and are identifiable using historical data. The behaviour may be categorised according to the pattern of the observed CM information, the failure times of components, or both. Stochastic filters are constructed for each contingency and we develop a Bayesian model that is used to recursively evaluate the probability that the observed CM information corresponds to each of the failure modes. The output from the individual filters is then weighted accordingly. Two scenarios are considered with the first involving a fixed but unknown underlying failure mode and the second catering for transitions between the failure modes over time. An example is presented using simulated data to illustrate the applicability of the methodology.

1. Introduction

CBM applications utilise CM information when scheduling maintenance and replacement activities for individual components. The components degrade stochastically over time under operational conditions and CBM models are used to reduce the occurrence of costly and untimely failures. A number of relevant models exist in the literature including proportional hazards models (Makis & Jardine (1991) and Vlok *et al*. (2002)), accelerated failure-time models (Cox & Oakes (1984)) and stochastic filters (Wang & Christer (2000) and Wang (2002)). The models are parameterised using data sets consisting of CM histories pertaining to analogous components. For the research documented in this paper, we are considering CBM scenarios where multiple failure modes exhibit themselves and are identifiable using historical CM data. We assume that the different failure modes display behaviour that can be categorised according to the CM output, the failure times of the components or both.

The methodology proposed in this paper involves constructing a stochastic filter for each of the defined failure modes. A given filter is used to recursively establish a conditional density for the residual life at each CM point under the relevant failure mode. An estimate of the residual life of a component is then defined as a weighted combination of the respective output from the individual filters. Firstly, we consider a situation where, the underlying dynamics (or failure mode) are assumed to be fixed and conform to one of the proposed models for the case. This model is for use when the behaviour can correspond to a number of distinct behavioural types and we are simply unaware which type the current component conforms to. An example considered in section 5 involves the modelling and estimation of the residual life of a component when the behaviour can correspond to one of two potential failure modes. The behaviour is assumed to manifest itself in the form of failure time clustering as demonstrated in figure 1. Separate models are established for each scenario and a recursive procedure is developed to determine, during the life of a component, which model the underlying dynamics conform to using both the age and the available CM history. We also consider the potential for the dynamics to evolve or fluctuate during the life of a component. We assume that at any given stage, the underlying failure mode conforms to one or more of the proposed models and that unknown transitions between the individual failure modes occur over time. The transition probabilities must be estimated from available data and are modelled using a Markov chain.

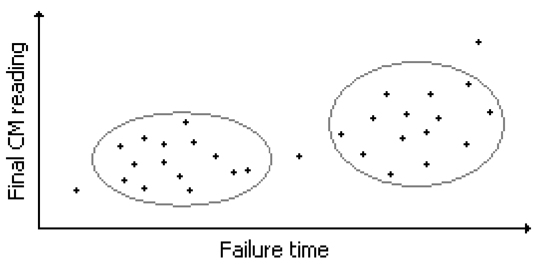

Figure 1. Illustrating the clustering of failure times when two different failure modes exist

2. An individual stochastic filter

In this section, we describe a stochastic filter designed to facilitate the residual life prediction for a component under the jth individual failure mode ($j = 1, 2, \ldots, r$). x_i is defined as the underlying residual life of a component at the ith CM point at time t_i . In addition, we observe a vector of CM parameters $\underline{y}_i$ and have the availability of the filtration or CM history $\Im_i = \{ \underline{y}_0, \underline{y}_1, \ldots \underline{y}_i \}$. From Wang (2002), the posterior conditional probability density for the residual life at the ith monitoring point given the history of CM information for a component is given by

$$p_{ji}(x_i \mid M_j, \Im_i) = p(x_i \mid M_j, \underline{y}_i, \Im_{i-1}) = \{p(\underline{y}_i \mid x_i, M_j, \Im_{i-1}) p(x_i \mid M_j, \Im_{i-1})\} / p(\underline{y}_i \mid M_j, \Im_{i-1}) \quad (1)$$

where, M_j represents the jth failure mode and $\underline{y}_i$ and $\Im_{i-1}$ are independent given x_i and we have

$$p(\underline{y}_i \mid x_i, M_j, \Im_{i-1}) \equiv p(\underline{y}_i \mid x_i, M_j) \quad (2)$$

The influence of the individual failure mode is reflected in the specification of the component parts of the relevant filter i.e. a density for the initial residual life $p_{j0}(x_0 | M_j, \Im_0) = p_{j0}(x_0 | M_j)$ and the density given by equation (2) representing the stochastic relationship between the monitored information and the underlying residual life. The second element of the numerator of equation (1) is derived as an updated version of the residual life distribution from the previous recursion, at time t_{i-1}, as

$$p(x_i \mid M_j, \Im_{i-1}) = p_{j,i-1}(x_{i-1} = x_i + t_i - t_{i-1} \mid M_j, \Im_{i-1}) / \int_{t_i - t_{i-1}}^{\infty} p_{j,i-1}(u \mid M_j, \Im_{i-1}) du \quad (3)$$

The denominator of equation (1) is established as

$$p(\underline{y}_i \mid M_j, \Im_{i-1}) = \int_0^{\infty} p(\underline{y}_i \mid x_i, M_j, \Im_{i-1}) p(x_i \mid M_j, \Im_{i-1}) dx_i \quad (4)$$

Using historical CM information and failure time data, the parameters of the individual stochastic filters are established for W relevant histories using the likelihood function

$$L(\underline{\theta}) = \prod_{d=1}^{W} \left(\prod_{i=1}^{n_d} p(\underline{y}_{di} | M_j, \Im_{d,i-1}) P_{jd,i-1}(x_{d,i-1} > t_{di} - t_{d,i-1} | M_j, \Im_{d,i-1}) \right) p_{jdn_d}(x_{dn_d} = T_d - t_{dn_d} | M_j, \Im_{dn_d}) \quad (5)$$

where a lower-case p represents a density function, an upper-case P represents a probability, $\underline{\theta}$ is the unknown parameters set and n_d is the number of monitoring points in the dth CM history ($d = 1, \ldots., W$).

3. Fixed failure mode

In this section, we discuss the weighted modelling approach for an individual component with a fixed underlying failure mode. The problem is essentially one of competing risks (Crowder (2001)) and we construct r different stochastic filters, each pertaining to an individual and distinct failure mode. The notation M_j represents failure mode j ($j = 1, 2, \ldots, r$) and with the availability of a set of past CM histories, the jth stochastic filter is parameterised using only those CM histories that correspond to failure mode j. The prior probability that the underlying dynamics of the CM process for a given component correspond to failure mode j is denoted as $p(M_j|\Im_0)$ and is easily estimated from historical data as

$p(M_j|\Im_0)$ = (Number of histories relevant to failure mode j) / (Total number of histories)

Considering a vector of condition monitoring parameters, $\underline{y}_i$, obtained at the ith discrete monitoring point at time t_i, we have

$$p(M_j \mid \Im_i) = p(M_j \mid \underline{y}_i, \Im_{i-1}) \quad (6)$$

as the conditional probability that the underlying dynamics of the current CM process correspond to failure mode j given the CM history available until that point in time. By the application of Bayes' law we obtain

$$p(M_j \mid \underline{y}_i, \Im_{i-1}) = \{p(\underline{y}_i \mid M_j, \Im_{i-1}) p(M_j \mid \Im_{i-1})\} / p(\underline{y}_i \mid \Im_{i-1}) \quad (7)$$

where, the initial probability $p(M_j \mid \Im_0)$ is assumed to be known and $p(M_j \mid \Im_{i-1})$ represents the probability that the underlying dynamics conform to failure mode j and is available from the previous recursion of the process. This is the means by which our best judgement regarding the actual underlying failure mode, and hence the residual life of the unit, is updated at each monitoring point. We also have

$$p(\underline{y}_i \mid M_j, \Im_{i-1}) = \int_0^{\infty} p(\underline{y}_i \mid x_i, M_j) p(x_i \mid M_j, \Im_{i-1}) dx_i \quad (8)$$

on the assumption that $p(\underline{y}_i| x_i, M_j, \Im_{i-1}) = p(\underline{y}_i| x_i, M_j)$, i.e. $\underline{y}_i$ is controlled by x_i and M_j only. The denominator of equation (7) is obtained by enumerating over all the possible scenarios as

$$p(\underline{y}_i \mid \Im_{i-1}) = \sum_{j=1}^{r} p(\underline{y}_i \mid M_j, \Im_{i-1}) p(M_j \mid \Im_{i-1}) \quad (9)$$

A weighted mean estimate of the residual life can be obtained as

$$\mathbf{E}[x_i \mid \Im_i] = \int_0^\infty x_i \, p_i(x_i \mid \Im_i)\, dx_i \tag{10}$$

where the weighted conditional distribution is simply

$$p_i(x_i \mid \Im_i) = \sum_{j=1}^{r} p_{ji}(x_i \mid M_j, \Im_i)\, p(M_j \mid \Im_i) \tag{11}$$

4. Failure mode transitions

To facilitate the modelling of underlying dynamics that can potentially vary over time as a component ages, we introduce the notation M_{ji} as being representative of the underlying dynamics conforming to failure mode j at the ith monitoring point. A time-invariant Markov chain is established with transition probabilities

$$a_{kj} = p(M_{ji} \mid M_{k,i-1}) \tag{12}$$

that correspond to the conditional probability that the underlying dynamics currently conform to failure mode j at the ith monitoring point given that they conformed to failure mode k at the previous monitoring point. The objective of the combined modelling approach with evolving dynamics is to establish the conditional distribution

$$p_i(x_i \mid \Im_i) = \sum_{j=1}^{r} p(x_i \mid M_{ji}, \Im_i)\, p(M_{ji} \mid \Im_i) \tag{13}$$

Both terms in equation (13) require some explanation. The first is established as

$$p(x_i \mid M_{ji}, \Im_i) = p(x_i \mid M_{ji}, \underline{y}_i, \Im_{i-1}) = \{p(\underline{y}_i \mid x_i, M_{ji})\, p(x_i \mid M_{ji}, \Im_{i-1})\} / p(\underline{y}_i \mid M_{ji}, \Im_{i-1}) \tag{14}$$

where, the relationship $p(\underline{y}_i | x_i, M_{ji})$ is available from the model specification and we have

$$p(x_i \mid M_{ji}, \Im_{i-1}) = \sum_{k=1}^{r} p(x_i \mid M_{ji}, M_{k,i-1}, \Im_{i-1})\, p(M_{k,i-1} \mid M_{ji}, \Im_{i-1}) \tag{15}$$

In this context, $p(x_i \mid M_{ji}, M_{k,i-1}, \Im_{i-1}) = p(x_i \mid M_{k,i-1}, \Im_{i-1})$ as the one step prediction of x_i is available from the previous recursion and is not dependent on the current model due to the lack of reliance on $\underline{y}_i$. We also have the reverse transition expression

$$p(M_{k,i-1} \mid M_{ji}, \Im_{i-1}) = \{a_{kj}\, p(M_{k,i-1} \mid \Im_{i-1})\} / \sum_{k=1}^{r} a_{kj}\, p(M_{k,i-1} \mid \Im_{i-1}) \tag{16}$$

and the denominator of equation (14) is established as

$$p(y_i \mid M_{ji}, \Im_{i-1}) = \int_0^\infty p(\underline{y}_i \mid x_i, M_{ji})\, p(x_i \mid M_{ji}, \Im_{i-1})\, dx_i \tag{17}$$

Now we consider the second term of equation (13). Assuming the initial probability that the underlying dynamics at the start of the CM process for a new component correspond to failure mode j, $p(M_j|\Im_0)$, is known, we again employ Bayes' theorem to recursively obtain

$$p(M_{ji} \mid \Im_i) = p(M_{ji} \mid \underline{y}_i, \Im_{i-1}) = \{p(\underline{y}_i \mid M_{ji}, \Im_{i-1})\, p(M_{ji} \mid \Im_{i-1})\} / p(\underline{y}_i \mid \Im_{i-1}) \tag{18}$$

where the constituent elements of the numerator are

$$p(\underline{y}_i \mid M_{ji}, \Im_{i-1}) = \int_0^\infty p(\underline{y}_i \mid x_i, M_{ji})\, p(x_i \mid M_{ji}, \Im_{i-1})\, dx_i \tag{19}$$

$$p(M_{ji} \mid \Im_{i-1}) = \sum_{k=1}^{r} a_{kj}\, p(M_{k,i-1} \mid \Im_{i-1}) \tag{20}$$

and the denominator is given by enumerating over the prediction available from all the potential models as

$$p(\underline{y}_i \mid \Im_{i-1}) = \sum_{j=1}^{r} p(\underline{y}_i \mid \Im_{i-1}, M_{ji})\, p(M_{ji} \mid \Im_{i-1}) \tag{21}$$

5. Example – fixed dynamics

In this example, we consider the modelling and estimation of the residual life of a component using vibration information when two potential failure modes are assumed to have been identified from relevant data in a scenario similar to that illustrated in figure 1. When the monitoring process commences for a new component, the underlying dynamics are assumed fixed but unknown, as described in section 3, and we develop two separate stochastic filters (filter 1 and filter 2) to represent each potential eventuality. The filters are developed using the same functional forms but are parameterised independently using relevant analogous component histories. The filters are then conducted in parallel and their respective output weighted according to the probability that the underlying dynamics correspond to the relevant failure mode. In this example, we simulate a cycle of data in accordance with each modelling formulation and investigate the ability of the prescribed methodology to track the appropriate underlying failure mode and the residual life of the component. The estimate of the residual life at each monitoring point is then compared with the estimate from a general stochastic filter (filter 3) that is developed and parameterised using all the available monitoring information, i.e. the histories are not classified according to any failure mode and are all grouped together for parameter estimation purposes. This is achieved by simulating a large number of cycles of CM data corresponding to failure modes 1 and 2 and parameterising a general stochastic filter (filter 3) using all the simulated output. We then compare the weighted output from filters 1 and 2 with the output from filter 3 to ascertain the benefit of the combined modelling approach for this particular scenario.

From equations (1) – (4), the filtering expression for filter j is

$$p_{ji}(x_i \mid \Im_i, M_j) = \frac{p(y_i \mid x_i, M_j)\, p_{j,i-1}(x_i + t_i - t_{i-1} \mid \Im_{i-1}, M_j)}{\int_0^\infty p(y_i \mid x_i, M_j)\, p_{j,i-1}(x_i + t_i - t_{i-1} \mid \Im_{i-1}, M_j)\, dx_i} \tag{22}$$

for $j = 1, 2, 3$. The constituent elements of filter j are the initial residual life distribution

$$p_{j0}(x_0 \mid M_j) = \alpha_j (\Gamma(\beta_j))^{-1} (\alpha_j x_0)^{\beta_j - 1} e^{-\alpha_j x_0} \tag{23}$$

which is defined as a Gamma distribution for each model but parameterised independently. Similarly, the distribution governing the conditional relationship between the observed vibration reading and the underlying residual life is taken to be Gaussian for all the filters as

$$p(y_i \mid x_i, M_j) = (1/\sigma_{ji}\sqrt{2\pi}) \exp\{-1/2\,((y_i - \mu_{ji})/\sigma_{ji})^2\} \tag{24}$$

where, for filter j, we have $\mu_{ji} = A_j + B_j \exp\{-C_j x_i\}$ as the expected vibration level at the ith monitoring point given a particular realisation of the underlying residual life. We assume that the standard deviation parameter is proportional to the vibration level as $\sigma_{ji} = d_j y_i$. The specified and estimated parameters of filters 1 and 2 are given in table 1.

Parameter	Filter 1	Estimate	Filter 2	Estimate
A	5	5.11	5	4.981
B	17.5	17.3	21	20.07
C	0.025	0.027	0.01	0.011
d	0.12	0.126	0.15	0.141
α	0.2	0.218	0.1	0.115
β	45	44.205	75	74.504

Table 1. The estimated parameters of filters 1 and 2

The expected CM paths for the average life corresponding to model formulations 1 and 2 are illustrated in figure 2. The general filter (filter 3) is constructed with the same forms as filters 1 and 2, given by equations (23) and (24), and the parameters are estimated using 100 simulated histories. 50 of the histories are generated according to failure mode 1 and 50 according to mode 2. The reasoning for this is that, for simplicity and to demonstrate the methodology, we develop a scenario in which both contingencies are equally likely, i.e. before the monitoring process begins, we have the initial probabilities $p(M_1) = p(M_2) = 0.5$.

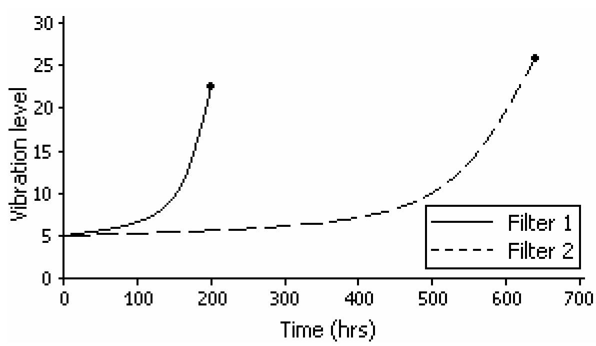

Figure 2. Illustrating the expected CM paths for failure modes 1 and 2

Using the CM histories simulated according to failure modes 1 and 2, the estimated parameters of the general stochastic filter (filter 3) are given in table 2.

Parameter	General Model
A	5.482
B	17.702
C	0.02
d	0.195
α	0.00778
β	3.266

Table 2. The estimated parameters of the general stochastic filter (filter3)

At the ith CM point, a closed form stochastic filtering expression is available for filter j as

$$p_{ji}(x_i \mid \Im_i, M_j) = \frac{(x_i + t_i)^{\beta_j - 1} \exp\{-\alpha_j (x_i + t_i) - \sum_{k=1}^{i} \phi_k(x_i, t_i)\}}{\int_0^\infty (u + t_i)^{\beta_j - 1} \exp\{-\alpha_j (u + t_i) - \sum_{k=1}^{i} \phi_k(u, t_i)\} du} \tag{25}$$

for which we define the function

$$\phi_k(u, t_i) = (1/2\sigma_{jk}^2)(y_k - A_j - B_j e^{-C_j (x_i + t_i - t_k)})^2 \tag{26}$$

An essential element in both the parameter estimation process and the determination of $p(M_j \mid \Im_i)$, see equations (6) – (9), is the distribution $p(y_i \mid \Im_{i-1}, M_j)$ given by equation (13). For the functional forms used in this example, we have

$$p(y_i \mid \Im_{i-1}, M_j) = \frac{\int_0^\infty (x_i + t_i)^{\beta_j - 1} \exp\{-\alpha_j (x_i + t_i) - \sum_{k=1}^{i} \phi_k(x_i, t_i)\} dx_i}{\sigma_{ji} \sqrt{2\pi} \int_{t_i - t_{i-1}}^\infty (u + t_{i-1})^{\beta_j - 1} \exp\{-\alpha_j (u + t_{i-1}) - \sum_{k=1}^{i-1} \phi_k(u, t_{i-1})\} du} \tag{27}$$

Failure times for the components are simulated using inversion on the initial life distribution $p(x_0)$ and the vibration readings are then generated at each CM point using inversion on the conditional density $p(y_i \mid x_i)$. We now simulate a case corresponding to each of the two failure modes and demonstrate the ability of the proposed methodology to track the appropriate mode and the underlying residual life. We compare the estimations of residual life and the prediction errors obtained using the combined weighted modelling approach (filters 1 and 2) with those obtained using the general stochastic filter 3 at each simulated CM point. The prediction error at the ith CM point is

$$e_i = ((x_i - \mathbf{E}[x_i \mid \Im_i])^2)^{1/2} \tag{28}$$

The mean-square error (MSE) about the simulated failure time is used as a criterion for comparing the weighted and general filters. Considering the weighted approach, the MSE attributable to each of the contributing filters is weighted according to the probability that each model provides an appropriate representation of the underlying dynamics for the particular component.

4.1 *Case 1*

For this first case, a cycle of CM data is simulated with the underlying dynamics corresponding to failure mode 1. The failure time for the cycle is 193 hours and figure 3 demonstrates the ability of the recursive process to track the appropriate mode according to equations (18) – (21) using equation (27) developed for this specific case.

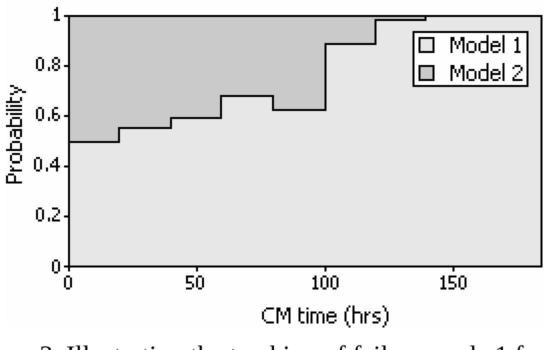

Figure 3. Illustrating the tracking of failure mode 1 for case 1

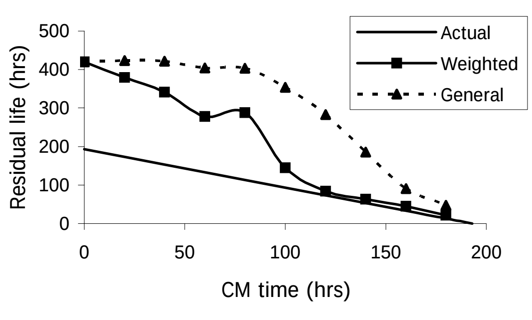

Figure 4. Comparing the residual life predictions obtained using the weighted approach (filters 1 and 2) and filter 3 for case 1

Figure 4 illustrates the tracking of the residual life at CM points throughout the life of the component. We compare the estimations of residual life given by the combined weighted modelling approach proposed in section 3 and the general filter.

Figure 3 clearly illustrates that the methodology tracks the appropriate failure mode for this particular case and figure 4 demonstrates a clear improvement with the residual life prediction capability of the combined modelling approach (filters 1 and 2) when compared with the general filter 3. In addition, the sum of squared errors for the weighted approach is 808.19 compared with 1776.6 for filter 3. The superiority of the combined approach is enhanced further by the MSE statistic of 345115 compared with 732541 for the general filter.

4.2 *Case 2*

For this second case, the CM process is simulated according to failure mode 2 with a failure time for the component of 651 hours. Figures 5 and 6 illustrate the tracking of the appropriate mode and the residual life respectively.

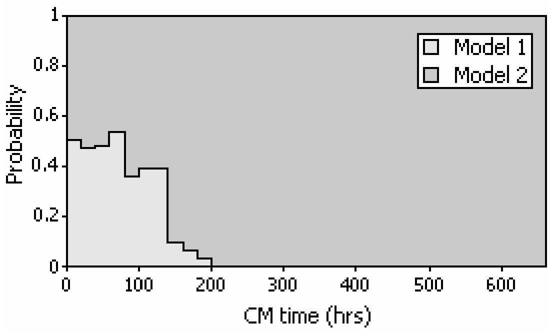

Figure 5. Illustrating the tracking of failure mode 2 for case 2

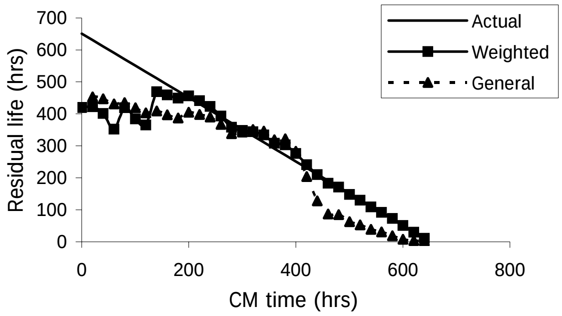

Figure 6. Comparing the residual life predictions obtained using the weighted approach (filters 1 and 2) and filter 3 for case 2

As with the first case, it is clear from figures 5 and 6 that the weighted approach (filters 1 and 2) tracks the appropriate failure mode quickly for this second case and that the estimates of the residual life are more accurate when compared with those obtained using the general filter 3. This conclusion is again confirmed by the fit statistics; the sum of squared errors is 1422.9 for the weighted approach and 2234.3 for the general filter and the MSE is 585240 for the weighted approach and 1050250 for the general filter.

6. Discussion

Cases 1 and 2 in the example have demonstrated that in some situations, it may be advantageous to group the available CM histories and construct a number of probabilistic stochastic filters to represent the specified contingencies (failure modes/types). The filters are then applied in parallel to new component CM information and the output from each filter weighted according to the recursively derived conditional probability that the filter is the appropriate representation of the current components underlying dynamics. The model introduced in section 4 incorporating transitions between failure modes will be explored in future research and a study is currently being conducted to test the application of the fixed mode methodology to an actual monitoring scenario.

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