Simon Jenkins wrote a piece for the Guardian on 7 August [1], which might charitably be described as a diatribe against things that he doesn’t understand. Some of his errors were pointed out in a letter from Paul Glendinning [2] but Jenkins’ ignorance regarding science and mathematics does not mean everything in the article is off the mark. There are quite a few ideas put forward in the article which I am entirely comfortable with; that science graduates should be articulate and able to explain themselves clearly, that modelling our maths education system on that of the old USSR or of China is a bad idea, for example. But nevertheless the article leaves a sour taste, I think because there are large numbers of influential people in the UK who have virtually no understanding of mathematics but whose ignorance results in them failing to see the importance of our discipline. I do not subscribe to the view that a degree in Politics, Philosophy and Economics from Oxford is the best basis on which to pontificate about the importance or otherwise of science or mathematics, which is somewhat of a worry when you realise that apart from Jenkins this group includes both our current prime minister and the leader of the opposition, as well as a host of other MPs.

Rather than dwell on this rather negative story, I would prefer to focus on much more significant events that took place later in August in Seoul, Korea, and in so doing celebrate some of the great things going on in mathematics today. I am referring to the major prizes that were awarded by the International Mathematical Union (IMU) at the 2014 International Congress of Mathematicians (ICM).

Four Fields Medals were awarded. These prizes, named after the Canadian Professor J.C. Fields who was secretary to the 1924 ICM held in Toronto, are arguably the most prestigious prizes in mathematics and are awarded every four years. For reasons which probably seemed sensible at the time but which now appear rather odd, candidates for the Fields Medal must be under 40 years of age on 31 December of the year preceding the award. Those honoured this year were Artur Avila, Manjul Bhargava, Martin Hairer and Maryam Mirzakhani (the latter being the first ever female recipient of a Fields Medal which is of course good news, though given that there have been 52 previous winners before this year does reinforce the view, that I have expressed before, that the world of mathematics research has a long way to go when it comes to issues of gender).

Avila’s research focuses on dynamical systems, bringing powerful tools from analysis to gain a deep understanding of such systems. His contributions, most of which involve collaborations with others, include a unified and comprehensive picture of the long-term behaviour of so-called unimodal maps (of which the archetypal example would be $x_{n+1} = \lambda x_n (1 - x_n)$), and important theorems relating to mixing in dynamical systems (to get a feel for this you can think of repeatedly applying exactly the same shuffle to a pack of cards and asking whether the result after many shuffles is completely mixed up). In the early 1980s Mark Kac offered 10 Martini’s to anyone who could solve a problem he had encountered involving the spectrum of a particular type of discrete one-dimensional Schrödinger operator. This problem, which arises in a number of physical contexts including the integer quantum Hall effect is particularly interesting mathematically because the spectrum of the operator depends fundamentally on whether one of the parameters in the system is rational or irrational. Avila and his coauthor solved the problem in 2005, though I have no idea whether anyone bought him 10 Martini’s, Mark Kac having died in 1984.

Bhargava is known primarily for his work in number theory. As a graduate student Bhargava did what few would contemplate and took it upon himself to read Gauss’ masterpiece, the Disquisitiones Arithmeticae. Gauss wrote a long section, full of detailed calculations, showing how one could combine two binary quadratic forms to produce a third and which Bhargava appears to have found unsatisfactory because he set about devising a better method. The real value in this is not that after 200 years someone has found out that Gauss’ approach was suboptimal, but that looking at the problem in a new way allowed Bhargava to generalise the results to higher degree forms. Prior to this mathematicians hadn’t even suspected that such generalisations existed. Some of Bhargava’s most spectacular results concern elliptic curves (e.g. at least 80% of elliptic curves have rank 0 or 1) but I will instead mention here a little nugget that stood out for me when I was looking into his work. When is a positive definite integral quadratic form (i.e. a quadratic form which always takes positive integer values) universal, i.e. under what circumstances does it take all possible positive integers as values? For example, $x^2 + y^2$ does not take the value 3 for any pair of integers $x$ and $y$, but Lagrange famously proved that all integers can be written as the sum of 4 squares so $x^2 + y^2 + z^2 + w^2$ is universal. Bhargava (with Jonathan Hanke) proved that there is a set of 29 positive integers, whose greatest element is 290, such that if a quadratic form takes these 29 values then it necessarily represents all positive integers.

Hairer works on nonlinear stochastic differential equations (SDEs). Differential equations have been the bread and butter of applied mathematics ever since they were first formulated in the 17th century. Just about every physical theory, from gravitation to quantum mechanics is based on them. In the 20th century mathematicians began to explore the complexities that arise when one or more of the terms in a differential equation is a random process. Such equations naturally model real situations where there is some form of noise in the system, such as fluctuating stock prices, or the behaviour of a polymer immersed in water due to the effect of the random collisions with water molecules. SDEs pose significant mathematical challenges. Most obviously they contain terms which cannot be differentiated and so even the word solution needs to be carefully defined. Distributions can be used to overcome some of these problems but since distributions cannot be multiplied together this makes nonlinear SDEs particularly troublesome. Hairer has created a new theoretical framework for looking at such problems which has advanced the field in a way no-one appears to have expected. The excitement around this development seems to be as much for the possibilities that have been opened up for others to build on as for the progress that Hairer has made to date.

Of all the four, Mirzakhani’s work is hardest (for me at least) to say something about that isn’t too technical. The citation from the IMU describes her work as bridging hyperbolic geometry, complex analysis, topology and dynamics which immediately gives one a sense of the enormous breadth of her contributions. One of her early results concerns closed geodesics...
on surfaces with genus at least two (i.e., surfaces with at least two ‘handles’). A geodesic is a curve which is the natural generalisation of a straight line to curved spaces; a geodesic connecting two nearby points will be the shortest curve connecting those points. Closed geodesics are just ones that come back on themselves (great circles are examples of closed geodesics on a sphere). It has been known for a long time that the number of closed geodesics whose length is less than some bound \( L \) grows exponentially with \( L \), but it turns out that if one restricts attention to only those geodesics that do not intersect themselves the situation is radically different. Mirzakhani showed that in this case the growth is of the order \( L^{6g-6} \) where \( g \) is the genus of the surface (assumed greater than 1). This work is just a small part of Mirzakhani’s contributions to the study of moduli spaces of Riemann surfaces, these spaces being geometric structures of dimension \( 6g - 6 \) about which there is still a great deal to learn.

The Fields Medals were not the only prizes to be awarded at the ICM, though there is not space here to comment on them all. Let me mention just one, the award of the Gauss prize for applications of mathematics to Stanley Osher. Perhaps Osher’s most significant work has been on the development of the so-called level set method used to describe how complex two-dimensional shapes evolve. The basic idea is to think of the boundary of the shape as the intersection of a changing curved three-dimensional surface with a horizontal plane. The film industry has taken this method and turned it into a hugely powerful tool for modelling two-dimensional representations of fluids and if ever you have wondered while watching the latest animated blockbuster as to how they make water or fire behave so realistically, Osher’s insight is the reason.

One of the striking things about this group of highly talented individuals is the geographical and cultural spread. Avila is Brazilian, but spends much of his time working in Paris; Bhargava works in the US but was born in Canada to parents who emigrated from India; Hairer is Austrian, went to university in Switzerland, and now works in the UK; and Mirzakhani was born and educated in Iran, though now works in the US. Mathematics is a global activity, and the best people travel large distances, uprooting and relocating families in the process, to find the environment that enables them to pursue their ideas most effectively. Returning, briefly, to Simon Jenkins’ article, perhaps the reason that the USA is so successful economically and yet performs poorly in league tables when it comes to maths and science education has something to do with their approach to immigration over the past 50 years.

Chris Linton FIMA

Zeeman Medal Winner: Marcus du Sautoy

The IMA and LMS are pleased to announce the award of the 2014 Christopher Zeeman Medal to Marcus du Sautoy of the University of Oxford for his contributions to the public understanding of the mathematical sciences.

Marcus du Sautoy has, since 2008, held the Charles Simonyi chair for the Public Understanding of Science at the University of Oxford. In fact he has been communicating mathematics to the general public for more than 20 years with considerable intensity and impact. Of particular note is his talent for engaging young people and inspiring them to follow mathematics.

His emphasis is to show the beauty of mathematics to wide audiences. This proves to be a powerful motivator for many people, young and old alike. Marcus has used all kinds of media to promote mathematics including appearing on and presenting radio and TV programmes, writing popular books and contributing to theatrical productions. He has shown particular skill in finding ways of explaining concepts of mathematics to audiences without specialist knowledge and who often believe the mathematical sciences are something for experts alone. Examples are programmes such as School of Hard Sums and Talk-Sport as well as news programmes on the World Service, BBC Radio 4, 5 Live and local radio. In 2006 Marcus become only the third mathematician to deliver the Royal Institution Christmas Lectures, which were widely regarded as exceptional.

Marcus has a particular talent for music and the creative arts. He took part in the Maestro at the Opera series on BBC2 in 2012. Though not about mathematics this appearance demonstrated to the viewing public that mathematicians have broad interests and talents. His play \( X \& Y \) which grew out of his collaboration with Complicité Theatre Company’s production of A Disappearing Number is an illuminating, thrilling work of theatre with mathematics genuinely at its core. His work with audiences exploring the mathematics in Mozart’s Magic Flute is similarly remarkable as a piece of mathematical communication.

Marcus’s popular books have also been exceptionally well received and have brought mathematics to wide audiences. His The Music of the Primes has been translated into 11 languages and sold more than 100,000 copies.

In addition to his work with public engagement Marcus has had a distinguished research career in the mathematical sciences, which has been marked by invitations to lecture at high profile international conferences, publication in leading journals and prestigious prizes. His academic research has straddled many areas of mathematics including group theory, number theory, model theory and algebraic geometry, with a primary focus on zeta functions of groups, functions first introduced by Grunewald, Segal and Smith in 1988. Marcus’s research has transformed the study of these zeta functions, revealing hidden depths and unexpected applications.

References
2. www.theguardian.com/politics/2014/aug/12/do-the-maths-education-choices