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t

ir travel is now common and significant advances have been made in the way aircraft are designed and operated since their first introduction. They are now much quieter and more efficient than their earlier counterparts and perform this feat whilst carrying greater payloads. The use of modern materials and manufacturing techniques play a significant part in this achievement; as do the methods of design, test, validation and certification. In this article focus is given to the gas turbine engine, which provides the thrust for the aircraft, and a brief overview of some of the mathematical techniques that underpin the monitoring of the engine through its operational life.

From its initial concept, through to design, manufacture, service operation and ultimately retirement from service, mathematics provides key tools to progress the gas turbine along its life-cycle journey. Forecasting techniques that incorporate regression models will be used to provide initial estimates of likely weight and therefore fuel burn efficiency for preliminary designs against proposed flight profiles and thrust requirements before an actual design is established. During the actual design process of its sub-system components consideration will be given to the specification and requirements of component geometry and operating conditions. This will require use of complex computational fluid dynamics to model the air flow through the core of the engine in order to achieve the expected compression capacities and efficiencies. In addition the gas loading and temperature profile will need to be understood to ensure that dynamic and steady-state stress predictions via finite element models are within the design and material properties tolerances of the component. The CAD tools used to help engineers define the physical shape and characteristics of compressor blades and other core components will incorporate trigonometry and geometric projection calculations to help the engineer visualise the final design.

The development of any new engine also requires rigorous testing to ensure that the design operates to model predictions and most importantly meets the required level for certification and safety. A significant amount of monitoring therefore takes place during engine testing using specialised instrumentation to ensure that the engine operates against its expected performance envelope and vibration characteristics. Mathematics again provides the tools to assist with this analysis through the use of signal processing techniques to translate the measured voltages from the transducers to a form that engineers can understand. In the case of vibration measurement this is likely to involve the Fourier transform which provides an efficient mechanism to convert a time-series signal into its frequency domain components allowing levels of resonant frequency points to be quantified and assessed.

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In-service health monitoring

The introduction provides only a glimpse into a fraction of the activity involved in the initial design and development of a gas turbine prior to entry into service. However, even when in routine production for revenue service the product continues to be monitored throughout its operation as part of what is often termed aftermarket care. This engine health monitoring service is provided to operators so that the first sign of any unusual behaviour can immediately be identified and rectified, preferably during routine maintenance, and hence with the objective of minimising service disruption. The utilisation of advanced diagnostic and prognostic monitoring tools is a significant component in achieving this goal.

Dedicated on-board instrumentation monitors the asset behaviour to provide an overview of key temperature and pressure conditions within the core of the engine along with the vibration response of the structure. Such data is recorded in a variety of formats, but routinely transmitted to the ground as a series of snapshot records at key steady-state points in the flight usually corresponding to take-off, climb and cruise. Such reports therefore provide a view of engine status represented as a vector of engine parameters at a specific point in time. Individual parameters may then be trended across a series of flights, adjusted to relevant ambient day conditions, to assess engine condition against expected deterioration rates. Key algorithms can also be applied to determine unexpected step changes in the data that may indicate early signs of incipient failure. This approach can provide a significant amount of information regarding fleet behaviour. For example, although aero-engines are designed to provide a given amount of thrust, airline operators may...
be advised of opportunities to take-off with less than the recommended rating. Such derated take-offs may be achievable depending on the take-off weight of the aircraft and ambient temperature conditions. Regular usage of low derate offers a benefit in terms of deterioration rate of the engine and therefore potentially extending the on-wing life of the engine before overhaul is required.

Figure 1 shows that simply applying the cumulative distribution function to the low-pressure shaft speed values (with z-axis values of N1 shaft speed shown as a percentage of maximum speed), reported at take-off, it is possible to determine that over a ten-year period the relevant operator has slowed take-off fan speed for a significant proportion of their fleet operation and hence reduced maintenance burden.

Simple statistical techniques, as in the example above, help to form an essential part of the health monitoring service. In addition to the example shown, this can be achieved in a variety of ways such as the use of control-chart type analysis or t-test comparison of an adjacent set of points over a series of flight reports. However, in cases where it is necessary to understand the structure of fleet service data, but taking account of several parameters, then more sophisticated methods based on multivariate techniques are employed to help provide visualisation of such high-dimensional complex data. Methods such as principal component analysis (PCA), Sammon’s mapping and NeuroScale are all viable techniques, each of which can be used in combination with other statistical and signal processing methods to gain insight into the data. For a detailed account of the theory behind these methods the reader is referred to other works such as [1, 2].

**Case study I: Identification of abnormality**

Having identified methods that are capable of handling multivariate data this article now focuses attention on how these techniques can support the development of models of normality. Such models offer a powerful mechanism for identifying unusual behaviour during service operation and avoid the main disadvantage with many data-driven fault identification approaches where it is necessary to have access to an adequate number of fault exemplars. As with most high-integrity plant, the gas turbine is extremely reliable and therefore such examples never occur in the quantities required for construction of a robust model. Novelty detection, on the other hand, is based on the premise that a bounded model of normality can be constructed from an abundance of normal data. Newly observed data can then be tested against this model in order to determine their relative novelty. Prior to being released into service, all new engines undergo a production pass-off test during which the vibration signature is assessed.

Figure 2 shows an example signature which is represented by approximately 100 points along the z-axis, corresponding to sampled engine acceleration speed. The units on the y-axis represent vibration amplitude, however, for the purpose of this example our focus is on the structure within the data and hence the techniques which follow will equally apply to any signature representation. Our goal is to derive a model of normality which captures the natural variability of such signatures from a representative sample (in this case just over 50). However, it is important to understand how such data is structured to ensure that such a model is a robust representation. The first problem to address here is the low number of examples for constructing a model in such a high-dimensional space. This can be addressed by summing adjacent points in each vibration vector to initially reduce the input dimension to say 10 dimensions. Obviously this is still too large to provide an effective visualisation via standard x/y plots or even 3D plots. For this example, we use principal component analysis as a mechanism to perform dimensional reduction down to two dimensions. Essentially we are using this technique to find the two main vectors, within the original data, the direction of which is the maximum variance within the data.
structure within the data and that it is indeed possible to describe
crincipal components. It will be seen that there is a high level of
responds to the measure of variance in the direction of its corre-
eigenvalue (one for each column of our input matrix) then cor-
responding eigenvector. Thus, the two eigenvectors that correspond
to the two largest eigenvalues will be used as the first two principal components. It will be seen that there is a high level of
structure within the data and that it is indeed possible to describe
this normal data by a series of clusters. In this case the $k$-means
algorithm has been used to locate four cluster centres, identified
by the green ‘+’ symbols. The corresponding green circles define
the boundary of each cluster at the 2-sigma limit. Thus our model
of normality can be defined by the overall boundary described by
these four clusters as opposed to the single boundary shown in red
which would result in a large number of false positive indicators.

In the example above the aim was to provide a model of nor-
mality based upon a series of normal vibration signatures. How-
ever, in order to generate such information several initial math-
ematical steps are required to extract such features from the ac-
celeration manoeuvre of the engine. Structural vibration is often
monitored from an accelerometer transducer which, with appro-
priate signal conditioning, provides such measurement in the
form of acceleration, displacement or velocity. Typically the ve-
locity measure is used to monitor engine structural response to
various sources of excitation. In the case of the gas turbine en-
gine, which comprises a number of rotating shafts, to enable the
compressor to extract energy out of the airflow, it is the very ro-
tation of these shafts that provide a source of excitation which is
manifested as force input to the engine structure at a frequency
(corresponding to shaft rotational speed. Therefore as the engine
accelerates through its speed range, the excitation frequency will
increase. The vibration response, to the main shaft excitation fre-
quency therefore varies according to resonant conditions of natu-
ral modes of vibration within the engine structure. This response
can be extracted by performing a fast Fourier transform on the ac-
celerometer time-series waveform to extract the frequency spec-
trum at different speed points throughout the acceleration ma-
oneuvre. The vibration profile of this signature is then simply
obtained by extracting the amplitude observed in each frequency
spectrum at the instantaneous shaft speed frequency.

In this example, we are using the vibration profile, from each
engine acceleration manoeuvre, as an input to the principal com-
ponent technique. However, it is also possible to use this tech-
nique for other parameter input types and hence the method is
widely used in a number of health monitoring applications (e.g.
engine core gas-path temperatures and pressures).

**Case study II: Empirical modelling of thermographic spectra**

As indicated above, the main benefit of the approach used in nov-
elty detection is that it takes advantage of available normal data to
help establish a reference model. However, there are many prob-
lems where it is desirable to obtain an empirical model that de-
pends on a large number of inputs, but with low example counts.
This poses a major challenge, since the robustness of the empir-
ical model depends on how well the model coefficients can be
fixed, or learnt, with respect to the training data and hence to
what extent the training data provides coverage of the problem
space. As a rule of thumb it is desirable to have at least ten times
as many training points as the model order complexity, assuming
these training points provide even coverage. The model order di-
rectly relates to the number of unknowns, or model coefficients,
that need to be derived during training.

In this example we consider how to derive a functional model
that maps thermal spectral readings to a temperature measure-
ment using non-linear regression. Coherent anti-Stokes Raman
spectroscopy (CARS) \[3, 4\] is a method of spectral temperature
measurement used in chemistry and physics with the ability to
monitor hot gases and flames in combustion systems where tem-
peratures can reach in excess of 2,500 K. This type of monitoring
would typically be performed as part of a rig test during an engine
development programme to assess the capability of a combustor
design. The temperature of the hot combustion gas is represented

![Figure 3: PCA projection of vibration data](image)

The upper graph in Figure 4 shows the spectral response from
different gas temperatures, with the dominant peak or that of its am-
plitude superimposed. As indicated above, the main benefit of the
approach used in novelty detection is that it takes advantage of
available normal data to help establish a reference model. However,
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design. The temperature of the hot combustion gas is represented

![Figure 4: Example CARS spectral response](image)
by a spectral profile containing 250 spectral lines. In this example, a series of 104 spectral measurements were taken at known temperature values covering a range of 250–3,000 K. An example of the spectral profile for two extreme temperature readings is given in Figure 4.

Figure 4: Example CARS spectral response

The upper graph in Figure 4 shows the spectral response from measuring a temperature of 250 K, whilst the lower figure indicates the spectral response from a temperature of 3,000 K. As is clearly seen the temperature cannot easily be interpreted from the frequency position of the dominant peak or that of its amplitude alone. It is also clear that temperature information may exist across all frequency bins and therefore our problem exists in 250 dimensions. With only 104 examples available it is very unlikely that we will be able to derive a standard regression model with 250 inputs and a single target value (i.e. temperature). However, one approach to this problem is to apply some method of dimensional reduction first. There are various options available, but taking advantage of the techniques introduced above, we find that application of principal component analysis is very effective at reducing this to a two-dimensional problem such that sufficient data is now available for both training and validation of a regression model.

Figure 5 shows that a very high level of variability in the data is retained projecting on to the first two principal components. In this example a multi-layer perceptron neural network was used as the non-linear regression model using three nodes in the hidden layer and thus requiring six coefficients to be determined during training of the model. According to the rule of thumb introduced above, this model requires a minimum of 60 training examples which is well within the data available from the experiment. The final result is shown in Figure 6, which shows the quality of fit for all estimated temperatures used in the experiment where the solid blue line indicates expected temperature, and the red points show the model’s predicted value for the corresponding thermal spectrum. The worst case error was found to be in the region of 50 K at the highest temperature value which corresponds to < 2% error of full scale and certainly sufficient as a method to provide temperature estimates from measured spectra.

Summary

This article has provided a glimpse into some of the mathematical and statistical techniques that support monitoring of gas turbine engines. Although emphasis has been given to clustering, dimensional reduction and regression techniques, it should be apparent to the reader that many other mathematical methods are utilised throughout the design, development, manufacture and operation of the gas turbine throughout its service life.

Notes

1. 100% engine speed corresponds to a nominal rated speed in rpm (typically 3900rpm), however the gas turbine is designed to operate, if required during take-off, much higher than this.

References