Initial Maths Test

You are not advised to ask anyone else to assist you in completing this as there will be a further written test as part of the Assessment Centre stage of the application process. However it is acceptable to use reference material if you need to.

This test is to be completed **online only** and not to be submitted separately from the application. This version is to enable you to complete the test prior to starting your application, as it cannot be saved part way through the submission process.

Please attempt all of the following questions but please note that you are not required to provide correct answers to all questions in order to progress to the next stage of the application process.

In each question just one of the statements (a) – (d) is true. Select one of these if you are confident of the answer or (e) if you are not sure.

1. Let \( A = \begin{pmatrix} 1 & 2 & -1 \\ -2 & -4 & 2 \\ 1 & 0 & 3 \end{pmatrix} \)

   *(Hint: You may find the Cayley-Hamilton Theorem helpful.)*

   a) \( A^9 = 4096 A \).
   b) \( A^{10} = 10,000 A \).
   c) \( A^{11} = 4,194,304 A^2 \).
   d) \( A^{12} = 1,000,000 A^3 \).
   e) Not sure.

2. Let \( g(\theta) = \sin(2\theta)\cos(\theta) \), and set \( K = g^{(2014)}(\pi/2) \), the 2014th derivative of \( g \) with respect to \( \theta \) evaluated at \( \theta = \pi/2 \).

   a) \( K \) is a rational number but not an integer.
   b) \( K < 1 \).
   c) \( K > 10^{1066} \).
   d) \( K \) is an integer whose final (units) digit is equal to 4.
   e) Not sure.
3. In the statements below, $C$ will be a simple closed curve containing the origin and symmetrical about both axes in the Cartesian plane. Furthermore, $R$ will denote a variable inscribed rectangle with sides parallel to the axes.

a) If $C$ is the circle $x^2 + y^2 = 1$, then the maximum area of $R$ is $2\sqrt{2}$.
b) If $C$ is the curve $y^2 + |x| - 1 = 0$, then the maximum area of $R$ is $4/3\sqrt{3}$.
c) If $C$ is the ellipse $4x^2 + y^2 = 1$, then the maximum area of $R$ is $1/\sqrt{2}$.
d) None of the preceding statements a), b) or c) are true.
e) Not sure.

4. A and B are playing a simple game with nine face-up tiles numbered 1 to 9. They take tiles alternately, and the first to collect three tiles that sum to 15 wins.

a) The first to play has a winning strategy.
b) The second to play has a winning strategy.
c) The first to play can force a draw.
d) The second to play can force a draw.
e) Not sure.

5. Two circular planar discs are slotted together with their planes at right angles to make a rigid three-dimensional structure; each disc has radius $r$ and is cut by a radial slot of length $s$ from a point on the circumference. Observe that through each point on a certain arc of points on the circumference of each of the discs there is a unique plane tangent to both discs, and this means that the rigid object can roll continuously along a horizontal plane. The value of $s$ has been chosen so that the object’s centre of gravity remains a fixed height $h$ above this plane as it rolls, which means that it moves smoothly, like a ball. (You may ignore the thickness of the discs and their slots.)

a) $s = r/(1 + \sqrt{2})$.
b) $r = (2 + \sqrt{2})s$.
c) $h = 2(\sqrt{2} - 1)s$.
d) None of the preceding statements a), b) or c) are true.
e) Not sure.

6. A projectile is launched from the origin $O$ with initial speed $u$ at an angle $\theta$ to the x-axis $Ox$. It is fired directly up an inclined plane represented by a straight line $OA$, which makes a positive angle $\alpha$ with $Ox$; you may assume that $\alpha < \theta < \pi/2$. Let $R_\theta$ denote the point where the projectile, moving under the force of gravity with downward acceleration $g$, hits the inclined plane. Finally, let $R_{\text{max}}$ denote the maximum value of $R_\theta$ as $\theta$ ranges between $\alpha$ and $\pi/2$.

a) $R_{\text{max}} = \frac{u^2}{g} (1 - \sin \alpha)$.
b) $R_{\text{max}} = \frac{u^2}{g} \cos^2 \alpha$.
c) $R_\theta = R_{\text{max}}$ when $\theta$ bisects the angle $AOy$ between the plane and the vertical.
d) $R_\theta = R_{\text{max}}$ when $\theta = \alpha/2$.
e) Not sure.
7. Let $V$ be a finite-dimensional vector space over the field of real numbers. Let $U$ and $W$ be subspaces of $V$ and let $S$ denote the set-theoretical union of $U$ and $W$.

a) The set $S$ is always a subspace of $V$.  
   b) The set $S$ is never a subspace of $V$.  
   c) The set $S$ is a subspace of $V$ if and only if $U = W$.  
   d) None of the preceding statements a), b) or c) are true.  
   e) Not sure.

8. Let $S$ denote the set of ordered pairs of integers $(m, n)$ satisfying the Diophantine equation $m^2 - 6n^2 = 1$. (The word “Diophantine” implies a restriction to integer solutions only.)

a) The set $S$ is empty.  
   b) The set $S$ is contains exactly one element.  
   c) The set $S$ contains more than 1 element but is finite.  
   d) The set $S$ is infinite.  
   e) Not sure.

9. Five students are sitting an exam at a row of 5 desks; their names are Alice, Bob, Charlotte, David, and Esther and they wear T-shirts coloured yellow, green, purple, red, and blue. They all have posts in their college maths society: Sabbatical Officer, Honorary President, Social Secretary, Treasurer, and University Committee Delegate, and the desks are lined up in that order.

- The student at desk 1 is wearing a red T-shirt
- The student with the blue T-shirt is sitting between students wearing red and green
- The student at desk 5 is wearing a purple T-shirt
- Alice is sitting next to David
- Esther is wearing a green T-shirt
- Bob is sitting between Charlotte and Esther
- David is sitting at desk 5

The Honorary President's T-shirt is

a) Green.  
   b) Yellow.  
   c) Blue.  
   d) Purple.  
   e) Not sure.

10. Consider this sequence:
3, 13, 1113, 3113, 132113, 1113122113, 311311222113, ...

The next term is

a) 13211321322113.  
   b) 13112122133122131.  
   c) 1311113111112121211113.  
   d) 31311133113132113111312211113.  
   e) Not sure.
11. We are interested values of the real number \( x \) for which the following inequality holds:

\[ x^3 - 4x^2 - 7x + 12 > |x - 3| \tag{I} \]

a) There is no unbounded interval of the x-axis on which the inequality (I) holds.

b) There exists an irrational number \( \alpha \) such that \( x \) satisfies the inequality (I) if and only if \( x < \alpha \) or \( x > 5 \).

c) If \( \frac{(3 - \sqrt{5})}{2} < x < \frac{(3 + \sqrt{5})}{2} \), then \( x \) satisfies inequality (I).

d) None of the preceding statements a), b) or c) are true.

e) Not sure.

12. The seasonal song ‘The 12 days of Christmas’ has twelve verses and begins as follows:

1. On the first day of Christmas my true love sent to me: A Partridge in a Pear Tree.

2. On the second day of Christmas my true love sent to me: 2 Turtle Doves and a Partridge in a Pear Tree.

... until, by the final verse, we see that

12. On the twelfth day of Christmas my true love sent to me: 12 Drummers Drumming, 11 Pipers Piping, 10 Lords a Leaping, 9 Ladies Dancing, 8 Maids a Milking, 7 Swans a Swimming, 6 Geese a Laying, 5 Gold Rings, 4 Calling Birds, 3 French Hens, 2 Turtle Doves and a Partridge in a Pear Tree

Over the 12 days a total of 364 presents are given. If Christmas lasted 120 days and the giving of 120 presents followed the same pattern, how many presents would be given altogether?

a) Fewer than 100,000 gifts.

b) Between 100,000 and 300,000 gifts.

c) More than half a million gifts.

d) 440,440 gifts.

e) Not sure.