Robust Adaptive Beamforming using the Domain Weighted Principal Components Analysis Technique

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Abstract

We describe the Domain Weighted Principal Component Analysis (DW-PCA) technique and its application to the narrowband robust adaptive beamforming problem. This is founded on a basic paradigm shift from one of noise cancellation to one of signal separation.

The DW-PCA technique offers a new and simple technique for data analysis that allows prior information to be included in a soft manner. This allows the avoidance of both the over reliance on, and the rejection of, prior information.

We demonstrate that the DW-PCA technique can operate as a robust adaptive beamformer, with performance that depends upon a tuning variable $\mu$. This performance can exceed that of other robust ABF techniques. In offering a method of using both the data and ‘softly’ incorporating the prior information, the DW-PCA technique offers performance that is startlingly distinct from other ABF techniques.

1. Introduction

Adaptive beamforming (ABF) is widely used, in conjunction with sensor arrays, for the purposes of interference or jammer suppression in diverse fields such as communications, radar, sonar and seismology. A number of powerful ABF techniques have been developed, but most implement some form of adaptive noise cancellation and depend, implicitly or explicitly, upon the signal of interest (SOI) being absent from the training data. As a consequence, conventional ABF techniques are highly susceptible to errors in underlying assumptions made about the environment, the sources, or the array geometry. Their performance is found to degrade significantly when the response to the array of the SOI is not known very precisely – this means it is not possible to synthesise SOI-free data. In order to alleviate the associated problem of adapting in the presence of the desired signal, significant effort has been devoted to developing more robust techniques for ABF.

The basic ABF technique is the look-direction constrained least squares algorithm, as used in the Minimum Variance Distortionless Response (MVDR) beamformer, Haykin (1991), for which the sample matrix inversion (SMI) algorithm can be exploited. Techniques developed to be more robust include: adding further linear (often derivative) constraints to the problem, leading to a Linearly Constrained Minimum Variance (LCMV) Beamformer, Scott & Mulgrew (1995), eigenspace based beamformers, adding diagonal loading to the sample covariance matrix, and a worst-case optimisation approach, Shahbazpanahi et al. (2003). All of these techniques require some extra decisions to be taken before the beamformer is calculated, e.g. the LCMV requires extra constraints to be selected.

The purpose of this paper is to introduce the concept of domain-weighted PCA (DW-PCA) and show that it offers a new approach to robust adaptive beamforming. It is based upon a shift of paradigm away from that of adaptive noise cancellation towards one of semi-blind signal separation. It is based upon one key observation, commonly observed during the second-order stage of a blind signal separation (BSS) algorithm.
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If the total power of the SOI, over all receiver channels, is significantly different from that of the various interfering signals, then applying Principal Components Analysis (PCA) produces one output channel in which the SOI is significantly separated from the interference. The DW-PCA algorithm utilises this observation; it uses information about the steering vector for the SOI to modify its received power so that it is distinct from that of the interferers. Then PCA is used to extract the SOI in a simple, effective and robust manner. In this way, prior information about the SOI is used in a ‘soft’ manner, offering robust performance.

2. Background

We utilise the classical instantaneous, stationary, and linear mixing model, i.e. the received data matrix $X$ can be modelled as follows:

$$X = AS + N = CPS + N \quad (2.1)$$

where the columns of $X$, $S$, and $N$ contain snapshots of the received data, original signals and sensor noise respectively. These snapshots are complex vectors, indexed by time, e.g. $X = [x(1) \ x(2) \ \ldots \ x(T)]$ and $x(t) = [x_1(t) \ x_2(t) \ \ldots \ x_n(t)]^T$ where $[\ ]^T$ denotes the vector transpose. $P$ is a diagonal matrix containing the powers of the individual signals. $C$ is constructed columnwise so that the $p$'th column of $C$, i.e. $c_p$, contains a unit norm vector corresponding to the pointing vector of the $p$'th signal.

The goal of the technique is to find a beam forming vector $w$, such that when applied to the data matrix it produces an output $z$ that consists mainly of the SOI (WLOG $s_1$):

$$z = w^H X \quad (2.2)$$

The main algorithmic tool utilised by the DW-PCA technique is PCA. This can be carried out by applying the singular value decomposition (SVD) to the data matrix $X$:

$$X = Q^H \Lambda V \quad (2.3)$$

where $Q$ is an nxn unitary matrix, $\Lambda$ is an nxn diagonal matrix containing the singular values and $V$ is an nxT semi-unitary matrix i.e. $VV^H = I$.

3. Domain Weighted PCA

The DW-PCA technique consists of three stages: domain transform; primary channel enhancement and power based separation. We describe each of these in turn for one specific formulation of the DW-PCA technique that uses prior information in the form of an estimated steering vector for the SOI, i.e. $\hat{c}_1$:

**Domain Transform:** This performs an energy-preserving, non-adaptive transformation that utilises the prior information to split the data matrix into primary and auxiliary channels. This specific formulation operates in a similar way to the GSLC, Griffiths & Jim (1982). The primary channel is formed by a quiescent weight vector, $u_1^H$, chosen to satisfy a constraint $u_1^H c_1 = 1$. The set of $n-1$ auxiliary channels are formed by the matrix $\tilde{U} \in C^{n-1 \times n}$, where the rows of $\tilde{U}$ form a basis for the null space of $u_1^H$, hence $\tilde{U}$ is semi-unitary. This stage outputs a transformed data matrix:

$$X_i = UX = \begin{bmatrix} u_1^H \\ \tilde{U} \end{bmatrix} X \quad (2.4)$$
Domain Weighted PCA

**Primary Channel Enhancement**: This modifies the primary channel by multiplying it by a parameter $\mu$. Adjusting this parameter should adjust the total power of the SOI in the transformed data matrix forming the output of this stage:

$$ Y = DX = \begin{bmatrix} \mu & 0 \\ 0 & I_{n-1} \end{bmatrix} UX \quad (2.5) $$

**Power Based Separation**: This stage consists of applying PCA to the output of the previous stage, $Y$; this is done by using the SVD to find a unitary matrix $Q$ such that $QY$ consists of decorrelated rows:

$$ Y_i = QY = QDUX \quad (2.6) $$

For robust ABF, the output of the DW-PCA technique is the beamforming vector $w^H$ defined as the row of the matrix product $QDU$ most closely correlated with the estimate steering vector of the SOI, $\hat{c}_1$.

4. **Exemplar Results**

The results of a single computer simulation experiment are presented here to demonstrate the performance of the DW-PCA technique and its differing performance as the parameter $\mu$ is varied. The simulation consisted of 500 samples of two QPSK modulated signals impinging upon a 20 element linear array. Both signals arrived at a power of 0dB, the SOI from $+30$ degrees relative to broadside and the interfering signal from a randomly determined angle between $-15$ and $-45$ degrees relative to broadside. The accuracy of the SOI steering vector estimate, $\hat{c}_1$, was varied by adding an error to the estimated angle-of-arrival of between $+0$ and $+10$ degrees, in increments of $+0.5$ degrees.

Five different algorithms were run on the data: a fixed beamformer using $w^H = \hat{c}_1^H$; an MVDR beamformer using the SMI algorithm to calculate $w$; $w^H = \hat{c}_1^H$; an MVDR beamformer using the SMI algorithm with diagonal loading to calculate $w$; the worst-cast optimisation technique of Shahbazpanahi et al. (2003); and the DW-PCA with four different $\mu$ values, $1.1, 1.2, 2$ and $5$.

The results in figures 1 and 2 show that for large errors in the estimation of the SOI pointing vector the DW-PCA technique can outperform the other techniques by up to $+6$dB, if the right value of $\mu$ is selected. This comes at a cost of marginally reduced performance when the SOI pointing vector estimate is accurate.

The asymptotic behaviour of the DW-PCA technique is that as $\mu \rightarrow 1$ the DW-PCA becomes equivalent to PCA and as $\mu \rightarrow \infty$ the DW-PCA becomes equivalent to the fixed beamformer.

5. **Conclusions**

The DW-PCA technique offers a new, and very simple, technique for robust ABF. It allows available information about the SOI to be included in a soft manner – enabling less accurate prior information to be used than other techniques. The approximate prior information is used to pre-emphasise the SOI before a PCA stage is used to find an output composed mainly of the SOI.

The specific form of the DW-PCA introduced in this paper uses the most common form of prior knowledge i.e. a steering vector for the SOI. The technique can be extended quite easily to incorporate other types of prior information, e.g. estimated jammer AOA’s but this is not discussed here.
FIGURE 1. SNIR varying with error in pointing vector for a variety of signal extraction techniques.

FIGURE 2. Top-left section of Figure 1.

REFERENCES


