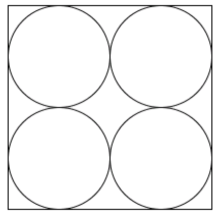
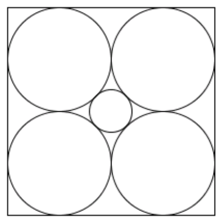
Circle Square

The diagram shows four circles of radius ¼ in a unit square.



A circle can fit in the gap in the middle.



What is the radius of the circle in the middle? What proportion of the original square is covered by the circle in the middle?

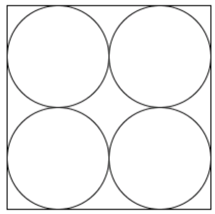
If you placed eight spheres of radius ¼ in a unit cube a sphere can fit in the middle. What is the radius of the sphere in the middle? What proportion of the original cube is covered by the sphere in the middle?

What happens in 4 dimensions?

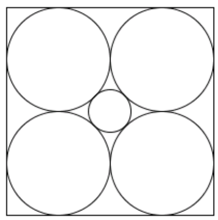
And in higher dimensions how big does the diameter of the hypersphere in the gap get and what proportion of the hypercube is covered?

Solution

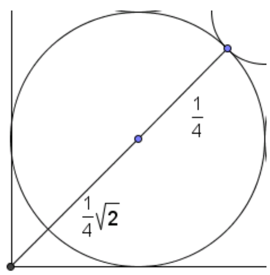
The diagram shows four circles of diameter ½ in a unit square.



A circle can fit in the gap in the middle.



This circle has diameter $\sqrt{2}-2\left(\frac{1}{4}\sqrt{2}+\frac{1}{4}\right)$



If you placed eight spheres of diameter ½ in a unit cube then you would find that a sphere of diameter $\sqrt{3}-2\left(\frac{1}{4}\sqrt{3}+\frac{1}{4}\right)$ would fit in the gap in the middle. This diameter is bigger than the diameter of the circle in the 2D version.

What happens in 4 dimensions?

And in higher dimensions how big does the diameter of the hypersphere in the gap get?

In 9 dimensions the diameter of the hypersphere in the gap is $\sqrt{9}-2\left(\frac{1}{4}\sqrt{9}+\frac{1}{4}\right)=1$ and in 10 dimensions the diameter of the hypersphere in the gap is $\sqrt{10}-2\left(\frac{1}{4}\sqrt{10}+\frac{1}{4}\right)=\frac{\sqrt{10}-1}{2}>1$ – the hypersphere doesn't fit inside the hypercube!