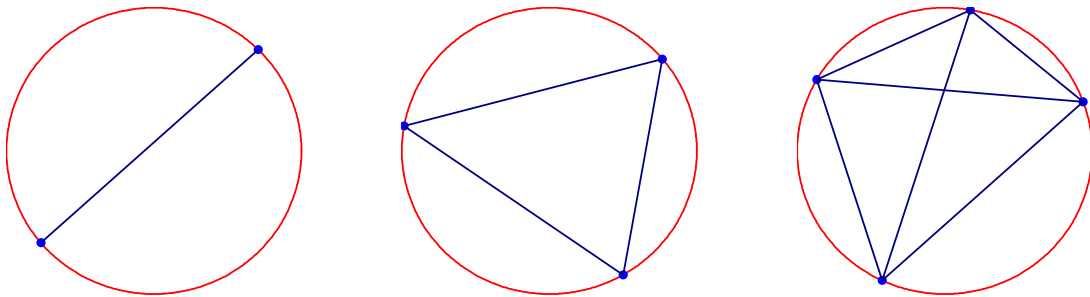


Circle Regions

Question

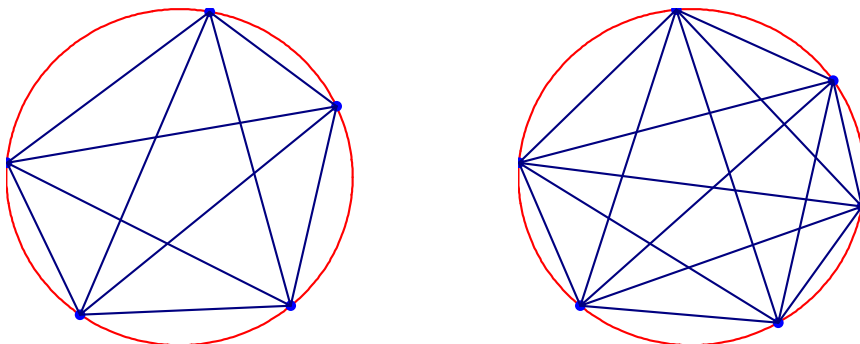
Put n points on the circumference of a circle. Join each point to every other one with a straight line. This cuts the circle into a number of regions. What is the maximum number of regions you can generate from n points?

For example, with two points you just get two regions, with three points you get four regions, with four points you get eight regions.



Solution

Did you just guess that the maximum number of regions keeps doubling with each additional point? That is, that the maximum number of regions with n points is given by 2^{n-1} ? Unfortunately, although this would give the right number for $n = 5$, where there are 16 regions, it breaks down for $n = 6$, where there is a maximum of 31, not 32 regions!



The general solution is given by:

$$\text{Maximum number of regions} = 1 + {}_n\mathbf{C}_2 + {}_n\mathbf{C}_4$$

$$\text{where } {}_n\mathbf{C}_m = \begin{cases} \frac{n!}{m!(n-m)!} & n \geq m \\ 0 & n < m \end{cases}$$

Try to derive this. Alternatively, visit http://en.wikipedia.org/wiki/Dividing_a_circle_into_areas.