Leapfrog

Adam and Brian decide to cycle from Adam's house to Brian's house 8 km away. Unfortunately Brian has left his bike at home! They decide to share Adam's bike, each alternately riding and walking. They agree that the rider will get off the bike when he is a distance of *h* km ahead of the walker and will continue on foot himself. Once the walker reaches the bike he will start to ride it, and continue riding until he is *h* km past the other, when he, in turn, will leave the bike and continue again on foot. This process will continue until they reach Brian's house. They set out together with Adam riding and Brian walking. Both of them walk at a uniform speed of 4 km/hr and ride at a uniform speed of 16 km/hr. What is the minimum time for the journey, as measured by the last one to reach the destination? If the bike changes hands twice during the journey what is the distance *h*?

(Problem prompted by the Leapfrog chapter in the book "Number Crunching" by Paul J. Nahin, Princeton University Press, 2011).

Solution

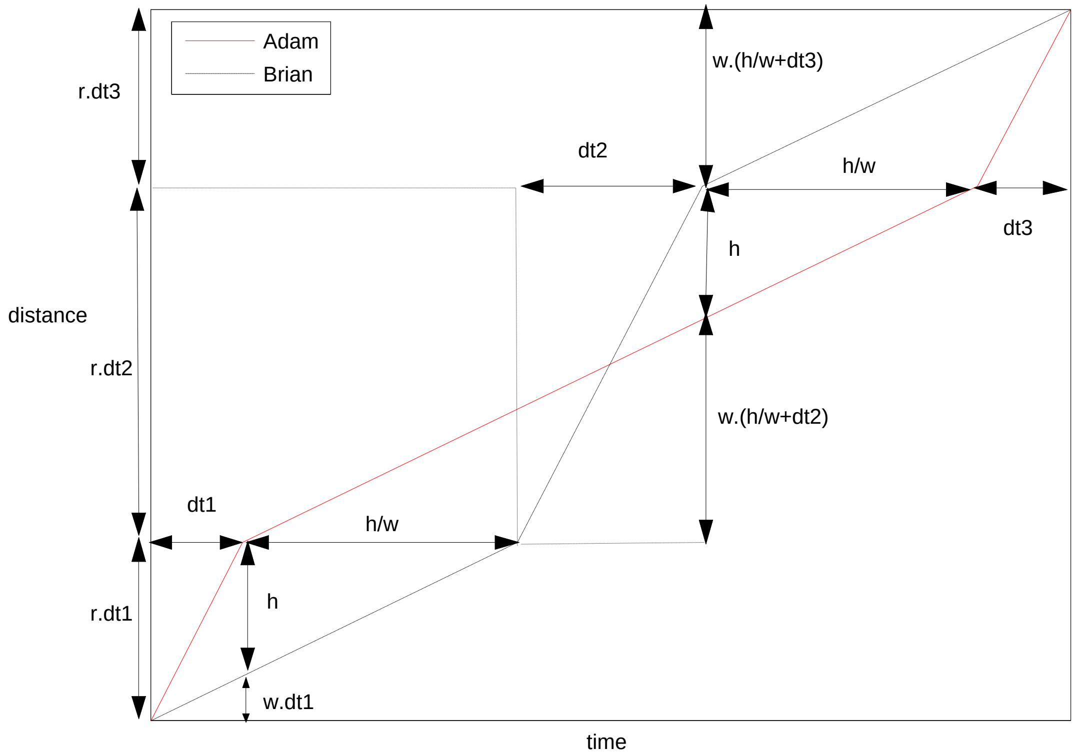
First, note that the minimum journey time requires both Adam and Brian to arrive *together*. To see this, assume that one arrives Z hours ahead of the other. If the faster one had reduced his bike distance and walked further he might have taken, say, X hours longer than he did (X<Z). The slower person could then have ridden the bike a little further and walked a little less, decreasing his time by, say, Y hours. By choosing the changes such that X+Y=Z, the two would arrive together, with a saving of Y hours over the previous time.

With this in mind, and using '*d*' for total distance, '*r*' for riding speed and '*w*' for walking speed, we can develop the following set of equations, using the plot below for guidance:

$$
\begin{aligned}
&r.dt1 + r.dt2 + r.dt3 = d \\
&r.dt1 = w.dt1 + h \\
&r.dt2 = w.dt2 + 2h \\
&r.dt3 = w.dt3 + h
\end{aligned}
$$

Here we have four equations that are easily solved for the four unknowns, *dt*1, *dt*2, *dt*3 and *h*. Note that the total time, *T*, say, is given by $T = dt1 + dt2 + dt3 + 2h / w$

With 'd', '*r*' and '*w*' as 8 km, 16 km/hr and 4 km/hr, respectively, then *T* = 1hr 15mins and *h* = 1.5 km.


Adam
Brian
r.dt3
w.(h/w+dt3)
dt2
h/w
dt3
h
distance
r.dt2
w.(h/w+dt2)
dt1
h/w
h
r.dt1
w.dt1
time