Dice Roll

Question

A standard dice is tossed until the total thrown exceeds 12. What is the most likely final total? For example, if it came up 3, 6, 1, 5 then the total would be 15.

Solution

All sequences of dice rolls are equally likely so we can do it by enumerating all possible sequences which end in each number.

But there is a short cut. Suppose a sequence stops with final result 14. The last throw can't have been a 1, or the sequence would have terminated on 13 in the previous round. So if the last dice throw had been one less, then it would have finished on thirteen. Thus for every sequence resulting in 14, there is one finishing on 13. So 13 must be at least as likely as 14, and in fact it is more likely because there are sequences finishing on 13 which do not arise in this way (for example any sequence ending in a 6. A similar argument shows that 13 is more likely than 15, 16, etc. So 13 is the most likely number.