Interview with Robert MacKay

Professor Robert MacKay FRS CMath FIMA FInstP is the Director of Mathematical Interdisciplinary Research and Director of the Centre for Complexity Science at the Mathematics Institute, University of Warwick. He took over as President of the IMA at the beginning of the year. He lists his research interests (I would say 'enthusiasms') as Dynamical Systems, Variational Problems, Hamiltonian Dynamics, Renormalisation of dynamical systems. These he applies to Physics, Engineering, Chemistry, Biology, Economics... I interviewed Robert MacKay at the Royal Society in June 2011.

Did you become interested in mathematics at Newcastle-under-Lyme School?

I would say I became interested well before that; my mother remembers worrying that I was late home from primary school at age four or five, and found me in the garden counting crocuses. I spent a lot of time calculating and read quite a bit of mathematics on my own. My parents gave me a book on calculus, Courant & Robbins *'What is Mathematics?'*, and two books on the history of mathematics. I used to go to the public library and take out books on topics like matrices. Perhaps the most significant factor in my school-age education was the private lessons that my mother organised for me with the former Head of Mathematics, Joe Sherratt, during my sixth form days.

What led you to Trinity College, Cambridge?

It was the dream of following Newton. Actually, I was admitted for natural sciences, because although I liked mathematics, I had no idea if I would be good enough for it, whereas I felt comfortable with physics and chemistry. But, on arrival, my Director of Studies recommended anyone with a mathematical bent who was thinking of concentrating on physics to take mathematics with physics in their first year and then switch to physics afterwards. So I did mathematics with physics and discovered I was good enough to do mathematics and that the mathematics curriculum included a lot of physics in any case, so I switched to mathematics at the end of the year.

Did you enjoy your time as an undergraduate?

I immensely enjoyed my time as an undergraduate. I made great friends, played a lot of music and learnt a lot of mathematics.

You went to Princeton University for your PhD in astrophysical sciences and the topic was area-preserving maps. Can you explain what that is and did you like America?

Yes, I wanted to work on a problem of potential social value that was nevertheless mathematically challenging, so I chose plasma physics, with a view to helping realise controlled nuclear fusion energy. I also wanted to see something different, in particular a different country. Nigel Weiss and Mike Proctor, from the University of Cambridge, told me in that case I should go to Princeton Plasma Physics Laboratory. I became interested in the basic question of motion of charged particles in a general magnetic field, rather than the axi-symmetric fields that tokomak theory usually assumed; or even simpler, following the magnetic field lines in a fusion device. The map from a poloidal section to itself generated by following the field lines for one toroidal revolution preserves magnetic flux. This can be regarded as a measure of area on the section; hence the problem reduces to iteration of an area-preserving map. The ideas that I helped develop are used regularly in the design of stellarators. Princeton provided me with a great opportunity to learn dynamical systems theory: in the plasma physics curriculum, from John Mather in the Mathematics Department, my Supervisor John Greene, and fellow students. I greatly enjoyed my time in the USA.

Did you get to see much of America while you were there?

Yes, the first summer I spent about seven weeks driving around. I started off with two friends from the same programme as me and we went hiking in the Cascades in the Washington State where they were from and then I left them there and carried on down to California, across Arizona, Utah, New Mexico and back.

Your career started at Queen Mary College, London and then has moved about between Warwick, France and Cambridge. Have your academic interests and natural curiosity driven this process?

I would say my career started already as a PhD student. I followed a combination of priorities and opportunities. I wanted to return to the UK after Princeton, and indeed was obliged to by the conditions of the Fulbright scholarship that had taken me there. John Greene's collaborator, Ian Percival from Queen Mary College, came to visit and was keen to hire me and obtained a postdoctoral research assistantship for me, so I took that up. But within a few months I received an invitation to spend a year at the Institut des Hautes Etudes Scientifiques near Paris, and a few months later David Rand from Warwick approached me about applying for a new lectureship there, which they then offered me. I ended up cutting short my postdoc position at Queen Mary College, spending seven months at IHES and then moving to Warwick where Rand and I created a great group. I turned down several tempting offers to leave Warwick until my wife was appointed a Professor of Physics at the University of Bourgogne, whereupon I took up a six-month position in mathematics there as a CNRS researcher followed by six months as a Professeur Invité. My hosts planned to obtain a permanent position for me as Directeur de Recherche in Pluridisciplinary Mathematics with the CNRS, but in the meantime David Crighton tempted us to Cambridge and in any case the CNRS decided I was unappointable! (they made up for this later by inviting me to serve on the Conseil Scientifique du département Mathématiques, Physique, Planète, Univers). Cambridge was wonderful for me, but my wife's future there was uncertain, so at the third time that Warwick tried to get us to return we said "yes".

You were involved in setting up the Nonlinear Systems Laboratory at Warwick in 1986 and then set up a Nonlinear Centre at Cambridge in 1995. Can you give the background to this and describe your interest in the subject?

Christopher Zeeman persuaded EPSRC (or SERC as it was then known) to fund research in nonlinear systems and so David Rand

and I put together an exciting proposal to combine experiment and theory on nonlinear systems, run visitor programmes and hire postdocs. It was a lot of work to set up but was a lot of fun and quite productive. When my wife and I moved to Cambridge we created a research centre there as a focus for various groups in the Mathematics Faculty active in nonlinear dynamics, to make an attractive environment for research students and to host long-term visitors and it worked well. I am interested in a wide range of theory and applications of nonlinear dynamics. Current projects include explaining cosmology without a big bang, deriving bifurcation diagrams for coupled oscillators, understanding singularities of robots, general theory of synchronisation of oscillators, making interesting examples of space-time phases, studying bifurcations of transition states, a mixing flow in a figure-eight knot complement, the dynamics of swarms, anomalous dispersal in mixing pipe flows, financial contagion, thermoeconomics, design of real-time electricity pricing systems, and robustness of near-integrability in zones of low shear.

Could you expand a little on the idea of cosmology without a big bang. In the 1940s, Hoyle, Bondi and Gold promoted the idea of steady state creation of the universe, in contrast to the big bang theory in which the universe began with a singularity. Can you explain where this alternative cosmology stands in relation to those?

Yes, standard cosmology is based largely on the Friedmann universes, which have a cosmic time coordinate. Einstein did the wonderful thing of removing the concept of absolute time and then cosmologists, including Einstein himself, put it back in, and this seems to me a big mistake. Obviously it was a good first step to think about the solutions of the Einstein equations for which space is essentially fixed except for its size, which can vary as a function of time. But it is rather limited compared with the realm of possible solutions of Einstein's equation. Of course, there is a lot of evidence that seems to fit the big bang story quite well. On the other hand, to save the big bang story, cosmologists have had to invent inflation and dark matter. I would also say dark energy, except for me that is Einstein's cosmological constant, which does not require a great explanation. With Colin Rourke, a topologist at the University of Warwick, we discovered a common interest in trying to find alternative models of the universe and it seems to me that de Sitter space (which is older than Friedmann universes; de Sitter space was proposed in 1917 and the Friedmann universe in 1922) has the ingredients of a much more sensible model, even though we don't propose that de Sitter space is an accurate model. In de Sitter space any typical pair of geodesics converge towards each other for a while and then diverge away from each other, whereas in the Friedmann universes all objects diverge from each other; if you go backwards in time they converge together. So in the context of something like de Sitter space, it is very exceptional for matter to have chosen to be on these mutually converging trajectories in backwards time. Now Hermann Weyl made a coherency postulate: he claimed that on philosophical grounds all matter is moving along some cosmic vector field that is orthogonal to some family of hypersurfaces, thereby defining an absolute time and making all matter converge together in at least one direction of time. I feel that this was a really big mistake, however much I may admire Hermann Weyl in many other ways. So we are trying to do without this concept of an absolute time and then there is no reason that everything should converge together in backwards time. We obtain a Hubble's Law in universes like de Sitter space, except you have to ignore the beginning period when the things are converging towards us because of course they are blue-shifted. But then we realised this could give an alternative explanation of the gamma-ray bursts which have been exciting cosmologists in the last fifteen years or so. In de Sitter space, for any object not converging to us in the past there is a first time at which it becomes visible to us and it starts with infinite blue-shift and very high intensity. The intensity drops off like $\frac{1}{t^2}$ and the blue-shift also drops off. Unfortunately the blueshift drops off rather slowly for most emitters, whereas observed gamma-ray bursts last a matter of minutes, but there is a parameter regime with arbitrarily short blueshift period. We are looking to see if there is some reason that selects them. Cosmologists explain gamma-ray bursts by all sorts of cataclysms but it seems to Colin Rourke and me that it may just be a kinematic effect of objects coming over our horizon.

What do you think are the most promising areas of research in nonlinear dynamics? What do you see as the greatest challenge in computational terms?

For promising areas, I would say the dynamics of systems with many components, and their control, particularly of probability distributions. One good computational challenge is to devise efficient aggregation schemes for systems with many components in order to achieve speed-up analogous to the fast Fourier transform algorithm.

Do quantum computers promise breakthroughs in the future if they reach the stage of being useful?

Proponents of quantum computers promise to be able to factorise enormous numbers quickly and solve many NP-hard problems easily; we will see.

You are the Director of Mathematical Interdisciplinary Research at Warwick. What is the purpose of this activity?

It is to stimulate research collaboration at Warwick between mathematicians and other scientists, via workshops, joint research students and projects. It has played a major role in the creation of various successful interdisciplinary research centres at Warwick like the Systems Biology Centre, the Centre for Scientific Computing, the Centre for Complexity Science, and the Centre for Discrete Mathematics and its Applications. It continues to be active in identifying opportunities and proposing strategy, like current ones in energy, medicine, and in social sciences.

Do you feel that there is enough attention paid to applying applications of mathematics found in one area of science to others? Are we missing opportunities here?

Yes, there is a lot of value in people who can see that the same mathematics, or very similar mathematics, could be applied somewhere else. It is something that I have enjoyed doing. For example, probability theorists have wonderful results about stochastic processes of many particles and I realised that this is essentially the mathematics that people in complexity science want when they want to make sense of ideas like emergence. Another example is that a bunch of physicists that I interacted with wanted to know whether there exist spatially localised time periodic solutions in their models of molecular crystals: 'discrete breathers'. I translated this into a problem in rather elementary functional analysis for the analysts. That had a big effect because it removed the doubts about whether what they were seeing numerically corresponded to a real solution or was just a long-lived thing. Another example of cross-fertilisation between different domains that we are involved in at the moment is the whole subject of contagion. It is naturally very well developed in the context of epidemiology, but similar ideas are relevant in financial contagion, which of course is extremely topical right now, also in the whole realm of computer viruses and the phenomenon of opinion dynamics.

Where does the concept of chaos theory fit into your work on nonlinearity?

Chaos is the popular term for sensitive dependence of deterministic dynamics on initial conditions. It is an important subfield of nonlinear dynamics. I have contributed to several aspects of the theory of chaos. One of which I am particularly proud was to provide a mechanical example of the purest form of chaos, called Anosov dynamics after a famous Russian mathematician who proved important results about this class of system. I call the example the triple linkage. It is a system whose configuration space was proposed by Thurston and Weeks as a simple illustration of the concept of a manifold but I realised its dynamics might be Anosov. With my student, Tim Hunt, we pinned down a parameter regime where indeed we could prove it. Another was to build on an incomplete idea of Poincaré about the effects of near collisions, to exhibit a much stronger form of chaos in the three-body problem than the one he identified.

You spoke at a conference entitled 'Harnessing Chaos'; can you explain what this term means?

In many application areas, chaos is seen as a nuisance to be avoided, but there are contexts in which it is useful, for example mixing fluids, random number generators, or controlling systems with a low expenditure of energy. Another from my plasma physics days is heating a plasma with waves: you want them to spread around.

Can you explain what quantum chaos is?

The question of quantum chaos is which features of the quantum mechanics of a system reflect the chaos that might be present in its classical limit system. One could think about a quantum mechanical version of the triple linkage for example. The farthest I got with quantum dynamics was developing a quantum analogue of 'discrete breathers': these are spatially localised time-periodic vibrations in networks of oscillators. Concrete examples are believed to occur in crystals of solid hydrogen, in the array of methyl rotors in crystals of 4-methyl pyridine and along the dimerised PtCl chains in an interesting charge transfer salt. I even got the Chemistry Department in Cambridge to make some of the latter for me and obtained some beam time at the ISIS Neutron Scattering Facility of the Rutherford Laboratory to try to detect spatially coherent vibrations but it turned out not be a suitable material for this type of experiment.

You are the Director of the Centre for Complexity Science at Warwick. Please explain what the purpose of this centre is and what you hope to achieve.

Following Yaneer Bar-Yam, I say complexity science is the study of systems with many interdependent components. The chief purposes of our centre are to achieve breakthroughs in the understanding, prediction, control and design of complex systems and to train a new generation of complexity scientists to solve the nation's and indeed the world's problems. The banking crisis (and now the Eurozone crisis), epidemics, reorganisation of the health service, questions about the climate, energy, transport and funding of universities, are all examples of complex systems where complexity science thinking is essential. In particular we must understand how to design socio-economic systems to incentivise individuals towards a generally agreed desirable social goal. But I do not have any easy solutions.

What do you think when you hear politicians and economists talking about the current worldwide financial problems and blaming particular local people, systems or events? Do you think they put too much emphasis on individual sources or situations?

In some cases you might be able to target an individual, in other cases maybe it is more a systemic problem and complexity science is particularly interested in seeing if we can understand systemic problems. I think financial bubbles have been happening since way back so they are not a new phenomenon and they will probably continue to happen unless we think of a suitable regulation system that would stop them being a systemic effect.

Complexity seems to be a subject that attracts extreme speculation and controversy. Given that there seems to be a gap in understanding of how basic organic compounds become living biological entities, are we going to see a mathematical description of emergence that might provide or describe a potential mechanism?

Indeed, I am afraid there is a lot of hot air generated under the banner of complexity science. Yet, in my opinion, a freedom of speech policy is appropriate for exploratory phases of a subject, as long as it is accompanied by an equal amount of critical thinking. There are also genuine advances in complexity science. In particular, I have a mathematical take on emergence. It does not identify the difference between life and inanimate, but here goes. I say that what emerges from a complex system are one or more probability distributions for state as a function of space and time. I call them 'space-time phases'. The amount of emergence in a space-time phase is its distance from the set of product distributions for independent units, in a suitable metric. I say a system exhibits 'strong emergence' if it has more than one possible space-time phase. This is the really interesting situation because often some of the phases exhibit strong collective effects.

Congratulations on being chosen as the next IMA President. You are involved in many learned societies. What do you see as the IMA's particular niche in the mathematical arena and what would you like to achieve as President?

Thank you. I think the IMA has a vital role to play as the professional society in the UK that represents mathematicians in all walks of life. I shall be pleased to lead it to yet greater influence. I hope to contribute to the issues of highlighting the impact of mathematics, prioritising resource allocation for mathematics research, planning the UK's involvement in the year of Mathematics of Planet Earth 2013, and supporting the initiatives for a greater appreciation for mathematics and skills in mathematics in schools.

There seems to be a perception amongst some members that the standard of mathematical ability in younger people has declined in recent years and, from the point of view of the Government support for various disciplines, mathematics seems to be coming off less successfully in comparison with some of the other subjects. What is your opinion on that?

I do not think there is a decline in mathematical ability; there are changes in the types of mathematical capabilities that are coming out of school these days. In particular school children are far more adept at using computers than in my day. In my day the closest I got to a computer at school was being invited to go to a college in Stafford for a two-day course where we punched things on paper tape and got printouts. It was really fun, I was very glad to have the opportunity, but things have moved on so I think children are developing different skills now. As far as the Government support for mathematics goes I think they recognise quite strongly the importance of mathematics for the socio-economic health of the country and indeed they are considering right now requiring mathematics up to age eighteen from all pupils. There is the question of what type of mathematics is appropriate and how this should be done, but I think the goal is to be praised.

What interests do you have outside of mathematics and your career?

I regret that most other interests I used to have like music, canoeing and politics have ended up being squeezed out. But I enjoy playing tennis, snooker, football, table tennis with my son, and family hiking in Switzerland. I still occasionally play the piano.❏

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