Radioactive Dice

Method 1

The probability of throwing a six is 1/6, so if we throw a large number, say N, of dice we are likely to lose a total of (1/6) N of them. That is, the change in number of dice after a single throw is $\delta N = -(1/6)\, N$ where the minus sign indicates loss. If we take a time δt to throw some of the dice, the rate of change of number of dice with time is $\delta N/\delta t = -(1/6)N$ (we now interpret the 1/6 as the fraction lost per unit time), or, taking the limit as $\delta t \rightarrow 0$, $dN/dt = -(1/6)N$. We can solve this to find N explicitly as a function of time: $N = N_0.e^{-t/6}$, where N_0 is the number of dice we started with.
If τ is the time it takes for N_0 to reduce to $N_0/2$ then $N_0/2 = N_0.e^{-\tau/6}$ or, after taking logs of both sides and a little rearrangement, $\tau = 6\ln(2)$, or $\tau \approx 4.16$.

Method 2

We can analyse this in a different way. If we start with N_0 radioactive dice, after one throw we will lose (1/6) N_0 of them and be left with (5/6) N_0 of them. If we throw these, we will lose (1/6)(5/6) N_0 of them and be left with (5/6)(5/6) N_0 or $(5/6)^2\, N_0$. That is, after **2** throws we are left with $(5/6)^{\mathbf{2}}\, N_0$. Repeating this reasoning we find that after **3** throws we are left with $(5/6)^3\, N_0$, and so on, so that after *t* throws we are left with $(5/6)^{\mathbf{t}}\, N_0$ radioactive dice. When $(5/6)^{\mathbf{t}}\, N_0$ equals $N_0/2$ we can replace *t* by τ, the half-life. By taking logs of both sides of the equation $(5/6)^{\tau}\, N_0 = N_0/2$ we find, after a little rearrangement, that $\tau = \ln(2)/\ln(6/5)$, or $\tau \approx 3.80$.

Questions to ponder

Both forms of reasoning show that the half-life of radioactive dice is roughly four throws. But the real questions for you to ponder are: 1. Why is the exact value different in each case? and 2. Which of the two is right?