A NEW VARIABLE STEP-SIZE LMS ALGORITHM WITH ROBUSTNESS TO NONSTATIONARY NOISE

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Abstract

A new variable step-size least-mean-square error (VSSLMS) algorithm is presented in this paper for applications in which the desired response contains nonstationary noise with high variance. The step size of the proposed VSSLMS algorithm is controlled by the normalized square Euclidean norm of the averaged gradient vector, and is henceforth referred to as the NSVSSLMS algorithm. As shown by the analysis and simulation results, the proposed algorithm has both fast convergence rate and robustness to high variance noise signals.

1. Introduction

The LMS algorithm has been extensively used in many applications as a consequence of its simplicity and robustness (Farhang-Boroujeny 1998; Sayed 2003). A key parameter in the design of LMS-based algorithms is the step size. As shown in Greenburg (1998), some normalization terms can be utilized to modify the LMS algorithm, so as to overcome the interference of nonstationary noise. One such modified LMS algorithm namely the sum method is discussed by Greenburg (1998). However, it is based on a constant convergence rate. Similar to in the case of the LMS algorithm, a variable step size algorithm is also necessary to obtain both fast convergence rate and small steady state MSE. In this paper, we propose a new variable step size algorithm, a NSVSSLMS algorithm, which is robust to high variance noise signals. In this algorithm, the step size is controlled by a normalized square Euclidean norm of the smoothed gradient vector. It will be shown that the proposed algorithm performs better than Greenberg’s sum method for the scenario when the noise signal is nonstationary.

The remainder of this paper is organized as follows: The proposed algorithm is described in Section 2. The analysis of the proposed algorithm in the context of stationary noise is introduced in Section 3. Simulations which confirm the analysis and the advantages of the proposed algorithm for nonstationary noise as compared with Greenburg’s method are shown in Section 4. Section 5 provides the conclusion.

2. Algorithm formulation

For the convenience of description, we formulate the LMS algorithm within the context of a system identification model. In this case the desired signal \( d(n) \) can be formulated as:

\[
 d(n) = x^T(n)w_{opt} + t(n)
\]  

(1)

where \( w_{opt} \) is the unknown filter which we wish to find, \( x(n) \) is the adaptive filter input vector, \( t(n) \) is the noise signal, \( n \) denotes the discrete time index and \( (\cdot)^T \) denotes the vector transpose operator. The output error of the system \( e(n) \) is:
\[ e(n) = d(n) - x^T(n)w(n) \]  

where \( w(n) \) is the vector of the adaptive filter weights. The proposed NSVSSLMS algorithm can be formulated as follows:

\[
\bar{g}(n) = \beta \bar{g}(n-1) + (1 - \beta) e(n)x(n) 
\]

\[
\mu_{NSVSS}(n) = P\|\bar{g}(n)\|^2_2 [L(\hat{\sigma}_e^2(n) + \hat{\sigma}_x^2(n))]^2 
\]

\[
w(n+1) = w(n) + \mu_{NSVSS}(n)e(n)x(n) 
\]

where \( \bar{g}(n) \) is the smoothed gradient vector, \( \beta \) is the smoothing parameter which is set to be very close to unity, \( P \) is a positive constant, \( \|\|_2 \) denotes the Euclidean distance, \( \hat{\sigma}_e^2 \) and \( \hat{\sigma}_x^2 \) are time varying estimations of the output error signal variance and the input signal variance respectively, \( L \) is the adaptive filter length, and \( \mu_{NSVSS} \) is the time varying step size.

### 3. Algorithm analysis

In this section we will give an approximate performance analysis for the proposed NSVSSLMS algorithm. For the convenience of analysis we make two assumptions:

A1. The input signal \( x(n) \) is a zero-mean white stationary Gaussian signal. The noise signal \( t(n) \) is a zero-mean stationary signal and independent with the input signal \( x(n) \).

A2. At steady state the excess error is much smaller compared with the noise signal, and therefore the error signal \( e(n) \) is approximately equal to the noise signal \( t(n) \).

Since the squared norm of the smoothed gradient vector \( \|\bar{g}(n)\|_2^2 \) is the key term for the proposed algorithm, we will give a steady state performance analysis for this term first. From (3) we have

\[
\bar{g}(n) = (1 - \beta) \sum_{i=1}^{n} \beta^{n-i} g(i) 
\]

assuming \( g(0) = 0 \) and denoting \( g(i) = e(i)x(i) \). The expected performance of the squared norm of the smoothed gradient vector can then be obtained

\[
E\{\|\bar{g}(n)\|_2^2\} = (1 - \beta)^2 \sum_{i=1}^{n} \sum_{j=1}^{n} C(ij) 
\]

where \( C(ij) \) is defined as

\[
C(ij) = E\{\beta^{n-i}g^T(i)\beta^{n-j}g(j)\} 
\]

When \( n \) approaches infinity, the term \( C(ij) \) in (8) approaches to zero if \( i \) or \( j \) is finite. So when we calculate \( E\{\|\bar{g}(n)\|_2^2\} \), the term \( C(ij) \) can be ignored when \( i \) or \( j \) is not infinite. The following analysis will only consider this term at steady state.

At first we consider \( C(ij) \) when \( i = j \). From assumption A2 we have

\[
g(i) \approx t(i)x(i) 
\]
Substituting this formulation into (8) and using assumption \( A1 \) we have

\[
C(ii) \approx \beta^{2n-2} L \sigma_i^2 \sigma_i^2
\]

(10)

where \( \sigma_i^2 \) and \( \sigma_i^2 \) are the variance of the noise signal and the input signal respectively.

When \( i \neq j \) similar derivation can be performed which yields

\[
C(ij) \approx 0 \quad (i \neq j)
\]

(11)

Substituting (10) and (11) into (7) we have

\[
E(\| \bm{g}(n) \|^2) \approx \frac{(1-\beta) L \sigma_i^2 \sigma_i^2}{(1+\beta)}
\]

(12)

Now let’s examine the steady state performance of the proposed algorithm. Since the term \([ L \left( \sigma_i^2 (n) + \tilde{\sigma}_i^2 (n) \right) ]^2 \) changes very slowly with stationary input and noise signals, we assume that it is a constant during the iteration. Taking the expectation from both sides of (4) and using equation (12) we have

\[
E\{\mu_{NSVSS}(\infty)\} = \frac{P(1-\beta) L \sigma_i^2 \sigma_i^2}{(1+\beta)[L(\tilde{\sigma}_i^2 (n) + \tilde{\sigma}_i^2 (n))]^2}
\]

(13)

if we assume that at steady state the step size of the proposed algorithm is very small, and \( \mu_{NSVSS} L \sigma_i^2 \ll 2 \), similar to the analysis of the LMS algorithm we can obtain

\[
J_{ex}(\infty) \approx \frac{P(1-\beta)L^2 \sigma_i^2 \sigma_i^4}{2(1+\beta)[L(\tilde{\sigma}_i^2 (n) + \tilde{\sigma}_i^2 (n))]^2}
\]

(14)

Since \( \tilde{\sigma}_i^2 (n) \approx \sigma_i^2 \), the following equation is obtained from (14)

\[
\lim_{\sigma_i^2 \to \infty} J_{ex}(\infty) \approx \frac{P(1-\beta)\sigma_i^4}{2(1+\beta)}
\]

(15)

It can be clearly seen from (15) that the EMSE obtained by the proposed algorithm will be independent of the noise signal \( t(n) \) when the variance of the noise signal is very large.

4. SIMULATION

In this simulation we will compare the performance between Greenburg’s method and the proposed algorithm within an adaptive noise canceller model. The input signal \( x(n) \) is a pseudo random, zero mean unit variance white Gaussian signal with a length of 100,000. The target signal \( t(n) \) is the first 100,000 samples of a speech signal which is available from http://www.voiptroubleshooter.com/open_speech/american.html, and the file name is “OSR_us_000_0016_8k.wav”. This target signal is scaled to make the average SNR over the entire observation 0dB. The unknown filter is obtained as follows:

\[
h(l) = e^{(-0.05l)} r(l), l = 1, \ldots, 100
\]

(16)

where \( r(l) \) is drawn from a zero mean unit variance white Gaussian sequence.

In this simulation the proposed algorithm is compared with Greenburg’s method with different step sizes 0.1 and 0.02. The initial step sizes and adaptive filter vectors of the proposed algorithm are set to be zero. The parameter \( \beta \) for the proposed algorithm is set to 0.999 to perform a sufficient smoothing operation. The parameter \( P \) in the proposed algorithm is set to 80. The parameter sets for the proposed algorithm are chosen to make its initial
convergence rate approximately equal to that of Greenburg’s method with a step size 0.1. The estimations $\hat{\sigma}_x^2$ and $\hat{\sigma}_z^2$ are obtained by using a first order filter, similar to that in Greenburg (1998). The evolutions of the EMSE curves for all the experiments are shown in Figure 1. The results are obtained over 200 Monte Carlo trials of the same experiment.

It is clear to see in Figure 1 that the proposed algorithm has a similar convergence rate of the EMSE with Greenburg’s method with a parameter 0.1 at the early state of the process. The EMSE of both methods converge to –20dB at about 3,000 samples. However, the EMSE of Greenburg’s method with parameter 0.1 fluctuates greatly with the variation of the target signal energy. The performance of Greenburg’s method with parameter 0.02 has a small EMSE and slightly fluctuation of the EMSE, but the convergence rate is very slow. The proposed algorithm has a fast convergence rate which is similar to Greenburg’s method with parameter 0.1, and a small EMSE which is close to that of Greenburg’s method with parameter 0.02. The proposed NSVSSLMS algorithm performs better than Greenburg’s method.

![The evolution curves of EMSE](image)

**FIGURE 1.** The evolution curves of EMSE with different methods

5. CONCLUSION
A new VSSLMS algorithm namely the NSVSSLMS algorithm has been presented in this paper. According to our analysis and simulation results the proposed algorithm performs better than Greenburg’s sum method with stationary input and nonstationary noise signals. Simulation show that this algorithm can obtain both a fast convergence rate and small EMSE with robustness to nonstationary noise signals.

REFERENCES