Modelling conflict: social choice theory and social welfare theory

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Abstract

We use social choice theory and social welfare theory in order to conceptualise conflict as a social process yielding a resolution outcome with a certain social value. Does the social process deliver social optimality? Social welfare functions are derived from Arrow's formulation of social choice. The social welfare function refers to utilities but, under certain circumstances, it can be re-expressed in terms of individuals' ideals. The social welfare of a situation depends on the welfare ceiling, the population sensitivity, the population variation and the deviation from the ideal. The social outcome can also be expressed in terms of individuals' ideals. Thus the social outcome function is comparable with the social welfare function. The deviation of the outcome from optimality depends on: the amount of systematic bias in power; the variation in ideals; and the correlation between the bias and ideal vectors. Social ideals are incorporated into the model. Misperception and misapplication of power press for an outcome different from the ideal. Social ideals and error can each have a progressive, neutral or regressive effect on social welfare. For a population with a group structure the deviation is the sum of the within-group deviation and the between-group deviation.

1. Introduction

Conflict is a social problem and seeking to resolve conflict is an attempt at social improvement. Social improvement is a major project in society - sometimes with social optimality as an ultimate vision. The theoretical assumptions underpinning such projects and visions have been particularly well-developed in the discipline of economics. Here we use social choice theory and social welfare theory in order to conceptualise conflict as a social process yielding a resolution outcome with a certain social value. We investigate the deviation of the outcome from social optimality.

2. Social optimality

In addressing the question of social optimality we shall follow in part the development offered by Jorgenson (1997, pp. 2-4, 63-74, 97-98). The type of situation in which we are interested involves: a set of individuals, say A; a set of social states, say X; a social ordering, say R, which is a complete, reflexive and transitive ordering of social states; and for each individual i, a preference ordering of social states, say P_i. We define $P = (P_1, \dots P_n)$ as a vector of individual preference orderings. We define L as the set of all possible such vectors. We define S as the set of all possible social orderings R. We define a social choice functional, say f, as a mapping from L to S, such that if f(P)=f(P') then R=R'. (Note that we are using a multiple profile approach to social choice rather than the single profile approach; Roberts, 1980).

What a social choice functional does is to offer us a way of defining social preference in terms of the set of preferences of individuals. However not all of the ways of doing this are satisfactory - we want social preference to be defined in a way which is 'reasonable' in relation to the set of individual preferences. Arrow (1963) suggested that it was reasonable that the relationship between individual preferences and social preferences should have the following properties: Pareto optimality, non-dictatorship, unrestricted domain, rationality and independence of irrelevant alternatives. (Arrow had also considered the property of positive association). Arrow showed that there were no social choice functionals which satisfied all five properties. Loosely speaking one can have either 'reasonable' dictatorship or non-'reasonable' non-dictatorship.

Arrow's result makes rather weak assumptions about individual preferences – he assumes only ordinal non-comparability. By making stronger assumptions we can perhaps broaden the class of reasonable social choice functions. Arrow's result still holds if cardinal non-comparability is assumed. However if cardinal comparability is assumed then there exists a class of social orderings which can be represented by certain types of social welfare functions: cardinal unit comparability yields utilitarian social welfare functions; and cardinal full comparability yields a class of social welfare functions which are the sum of two components, an average of the individual welfare functions plus a measure of dispersion in individual welfare levels (Jorgenson, 1997, pp. 3, 63-67). Further assumptions yield other classes of welfare functions (Jorgenson, 1997, pp. 3, 67-72).

Here we confine our attention to the utilitarian social welfare function. In equation (2.1) below, x is a social state, U is the social welfare function based on the vector $\underline{u}(x)$ of individual welfare functions $u_i(x)$ for individuals i=1, … N, and the 'welfare weights' w_i are constants with $\Sigma_{i=1}^{N} w_i=1, w_i \geq 0$.

$$U[\underline{u}(x)] = \Sigma_{i=1}^{N} w_i u_i(x) \qquad (2.1)$$

The set of social states may be discrete but here we make the assumption that the set of social states is a subset of a multi-dimensional real space. We say a utility function is separable if it can be expressed as the sum of functions, with each function dependent on just one dimension of the social state space. There are many situations where the utility functions are not separable. However here we focus on the simpler case of separable utility functions.

In the additive version of multi-attribute utility theory, the utility of an option is a weighted sum of the utilities of its attributes, the weights being the sensitivities of the dimensions. This has been found to be quite a robust model. (Borcherding et al., 1995; Diederich, 1995). Suppose the attributes are labelled j=1, …M. The social state $\underline{x}$ has co-ordinates $\{x_j\}$; and w_{ij} denotes the welfare weight attached to attribute j by individual i.

$$u_i(\underline{x}) = \Sigma_{j=1}^{M} w_{ij} u_i(x_j) \qquad \text{with } \Sigma_{j=1}^{M} w_{ij} = 1, w_{ij} \geq 0 \qquad (2.2)$$

Substituting this in equation (2.1) we can obtain two equivalent expressions for the social welfare function, either as the weighted sum of the individual welfares $u_i(\underline{x})$ or as the weighted sum of the social welfares U_j generated by each dimension j.

$$U[\underline{u}(\underline{x})] = \Sigma_{i=1}^{N} w_i\, u_i(\underline{x}) \qquad (2.3)$$

$$U[\underline{u}(\underline{x})] = \Sigma_{j=1}^{M} W_j\, U_j \qquad (2.4)$$

where $U_j = \Sigma_{i=1}^{N} a_{ij}\, u_i(x_j)$ and $W_j = \Sigma_{i=1}^{N} w_i\, w_{ij}$; $a_{ij}=(w_i w_{ij}/W_j)$; and $\Sigma_{i=1}^{N} a_{ij}=1$.

This follows from $U[\underline{u}(\underline{x})] = \Sigma_{i=1}^{N} w_i \Sigma_{j=1}^{M} w_{ij} u_i(x_j) = \Sigma_{j=1}^{M} \Sigma_{i=1}^{N} w_i w_{ij} u_i(x_j)$.

Now consider just one dimension, x. Suppose that, for individual i, the utility on this dimension is a single peaked function $u_i(x)$, attaining a maximum when $x=x_i$. We refer to x_i as the (personal) ideal state of the individual. So utility decreases with distance either side of the ideal state.

THEOREM 1. *If the individual utility functions are differentiable and single-peaked then the social welfare is single-peaked and has a maximum – at the 'welfare ideal' - within the interval [a,b] where a is the minimum and b is the maximum of the individual ideals.*

PROOF
If the $u_i(x)$ are differentiable and single-peaked then the $u_i'(x)$ are decreasing functions of x. Using $U[\underline{u}(x)]=\Sigma_{i=1}^{N} w_i u_i(x)$, we differentiate with respect to x to obtain: $U'=\Sigma_{i=1}^{N} w_i u_i'(x)$. So U' is a decreasing function of x. For $x<a$ all the $u_i(x)$ are positive and for $x>b$ all the $u_i(x)$ are negative. So $U'(a-\varepsilon)>0>U'(b+\varepsilon)$ for all $\varepsilon>0$. So there exists x^* such that $U'(x^*)=0$ where $a \leq x^* \leq b$.

It is possible to construct a quadratic Taylor approximation to the utility function near the ideal. Here we shall simply assume that the utility function *is* quadratic. In equation (2.5) below, the parameter c_i represents the satisfaction ceiling and the parameter m_i represents the sensitivity of utility to changes in the social state.

$$u_i(x) = - m_i(x-x_i)^2 + c_i \qquad m_i \geq 0 \tag{2.5}$$

In this situation the generalised utilitarian function in the form (2.1) above can be re-expressed in terms of the ideals of the individuals in the population.

THEOREM 2. *Suppose that the set of social states is one-dimensional and that each individual has a quadratic utility function. The welfare ideal x* is given by equation (2.7). The social welfare of a situation depends on the welfare ceiling C, the population sensitivity m≥0, the population weighted variation V and the deviation D of the situation from the welfare ideal. (Here each individual is assumed to have the same sensitivity m.)*

$$U[\underline{u}(x)] = - m (V + D^2) + C \tag{2.6}$$

$$x^* = \Sigma_{i=1}^{N} w_i x_i \tag{2.7}$$

$$V = V(\{x_i\}) = \Sigma_{i=1}^{N} [w_i(x_i-x^*)^2]; D = (x-x^*); C = C(\{x_i\}) = \Sigma_{i=1}^{N} w_i c(x_i)$$

THEOREM 3. *Suppose that each individual has a quadratic utility function with individual sensitivity m_i, then equation (2.6) still applies but with:*

$$m = \Sigma_{i=1}^{N} m_i w_i$$

Where the weights are now $\omega_i = m_i w_i / \Sigma_{i=1}^{N} m_i w_i$, with $\Sigma_{i=1}^{N} \omega_i=1$; $x^* = \Sigma_{i=1}^{N} \omega_i x_i$; $V(\{x_i\}) = \Sigma_{i=1}^{N} [\omega_i(x_i-x^*)^2]$; and $C(\{x_i\}) = \Sigma_{i=1}^{N} w_i c(x_i)$ as before.

THEOREM 4. *It follows that, in the multidimensional case, the social welfare of a situation involves the above parameters for each dimension.*

$$U[\underline{u}(\underline{x})] = \Sigma_{j=1}^{M} W_j\, U_j = \Sigma_{j=1}^{M} W_j\, [\, - m_j \, \{ V_j + D_j^2 \} + C_j] \qquad (2.8)$$

Which reduces to equation (2.7) if $W_j=1$ and $m_j=m$; with $V=\Sigma_{j=1}^{M}V_j$; $(x-x^*)^2=\Sigma_{j=1}^{M}D_j^2$ and $C=\Sigma_{j=1}^{M}C_j$

COROLLARY OF THEOREM 2. For a given population, social welfare U is maximised and is at the level, -mV+C, when the social state is identical with the welfare ideal, $x=x^*$ (thus justifying the phrase 'welfare ideal'). If in addition all the population share the same ideal, $x_i=x^*$ and so V=0, then the social welfare is simply $C(\{x_i\})$ … and if, further, this shared ideal maximises C, $x_i=x(C_{max})$, then the social welfare U is maximised at the satisfaction ceiling C_{max}.

PROOF OF THEOREM 2
$U[\underline{u}(x)] = \Sigma_{i=1}^{N} w_i\, u_i(x) = \Sigma_{i=1}^{N} w_i\, [-m(x-x_i)^2+c(x_i)] = - m\, \Sigma_{i=1}^{N} w_i\, [(x-x_i)^2] + C(\{x_i\})$
We now consider the first of the two terms on the right of this equation.
Note that $(x-x_i)^2 = (x-x^*+x^*-x_i)^2 = (x-x^*)^2+(x^*-x_i)^2 + 2(x-x^*)\,(x^*-x_i)$. We have:
$\Sigma_{i=1}^{N} [w_i(x-x^*)^2] = (x-x^*)^2$.
$\Sigma_{i=1}^{N} [w_i(x^*-x_i)^2] = V(\{x_i\})$, the 'population weighted variation'.
$\Sigma_{i=1}^{N} [w_i(x-x^*)\,(x^*-x_i)] = (x-x^*)\, \Sigma_{i=1}^{N} w_i(x^*-x_i) = 0$.

Note that the Pareto-optimal set is the minimal convex hull containing all the ideals and that the welfare ideal is in the Pareto-optimal set.

As a prelude to the next section, let us recap. We have used a set of individual preferences in order to define a social ordering of social states and thus to identify a social state the status of which is that it is most socially preferred. More strongly, we have used a set of utility functions in order to define a social welfare function and used this to identify the welfare ideal and its associated level of welfare. In order to reach Theorem 4 we have had to assume that the utility function is separable, single-peaked and quadratic. With these assumptions the maximum social welfare is, respectively, separable, occurs in the convex hull of the individual ideals and occurs at the welfare-and-sensitivity-weighted mean ideal.

3. The social process

We now consider the social process. By analogy with the previous paragraph, we can use a set of individual preferences in order to define a social ordering of social states and thus to identify the social outcome. More strongly, we can use a set of utility functions in order to define a social outcome function and use this to identify the social outcome. We define a social outcome function, say g, as a mapping from L to X, that is from the set of vectors of individual preference orderings to the set of social outcomes. This function too we wish to define in a reasonable way. To do this we continue with our assumption that the set of social states is a subset of a multidimensional real space. We suppose that the social outcome depends on the set of individual utility functions. Denoting the set of parameters of these functions by A, we have $x=g(A)$. Each individual's ideal is a parameter and so we have $x=g(\{x_i\},A\backslash\{x_i\})$. We now restrict our attention to situations with the following characteristics: (i) g is an increasing function of each of the x_i; (ii) x does not depend on any of the other parameters of the utility function; (iii) if all individuals have the same ideal then the outcome will be that ideal; and (iv) g is a linear function –or can be approximated by a linear function.

THEOREM 5. *Suppose that $\partial x/\partial x_i \geq 0$; $g(\{x_i=a\})=a$ for all a; and g is a linear function of the $\{x_i\}$. Then the social outcome is the power-weighted mean ideal:*

$$x = \Sigma_{i=1}^{N} w'_i x_i \qquad \text{where } \Sigma_{i=1}^{N} w'_i=1,\ w'_i \geq 0 \tag{3.1}$$

PROOF
If g is linear then $x=g(\{x_i\})=w+\Sigma_{i=1}^{N}w'_i x_i$. Each $w'_i=\partial x/\partial x_i>0$. For each a, $a=w+\Sigma_{i=1}^{N}w'_i a$. So $w=0$ and $\Sigma_{i=1}^{N}w'_i=1$.

NOTES*:*
(i) Linear power equations such as that above have been discussed by Allen (1992), Braun (1990), and Stokman and Zeggelink (1996), the latter including an empirical study.
(ii) In general the social outcome of the above-defined process is not the median and so is not the result of a democratic system of pair-wise voting. (See, for example, Rubinfeld, 1987, p. 602). Stokman and Zeggelink (1996) discuss their use of the mean as opposed to the median. See also Ma and Weiss (1995) on the mean voter theorem.
(iii) The social outcome is in the convex hull of the set of ideals and hence, assuming single-peaked utility functions, in the Pareto-optimal set.
(iv) Here we do not consider the dynamics of the social process except to note that one approach would be to have $dx/dt=\Sigma_{i=1}^{N} w'_i(x-x_i)$. This dynamics would then have an equilibrium at $x=\Sigma_{i=1}^{N} w'_i x_i$.

4. The deviation of the social outcome from social optimality

We now have accounts of the social welfare function and of the social process. Moreover, under certain assumptions, both the welfare ideal x* and the social outcome x can be represented as the weighted sum of individuals' ideals – see equations (2.7) and (3.1) above. This makes it straightforward to find an expression for the deviation D of the social outcome from optimality.

As a preliminary to obtaining this expression consider the distribution of individuals' ideals. We use μ_x and σ_x to refer to the mean and standard deviation of this distribution. We introduce the notion of the bias of power for each individual $b_i=(w'_i-w_i)$, in other words the extent to which the individual's power exceeds their weight in the social welfare function. We use μ_b and σ_b to refer to the mean and standard deviation of the distribution of bias over the population. Note that $\mu_b=0$ (because $\Sigma(w'_i-w_i)=1-1=0$). The correlation between ideals and biases is ρ_{xb}. The covariance between ideals and biases is $cov(b_i,x_i)=\Sigma b_i x_i/n$ because $\mu_b=0$.

We now introduce and motivate the concept of the bias magnitude b^. Consider a population with σ_b as the standard deviation of the bias. Now suppose that we have a new population consisting of n replications of the original population. All the weights and hence all the biases will be reduced by a factor 1/n. Thus the bias can be eliminated by arbitrarily replicating the population. To overcome this problem we define the bias magnitude, $b\hat{}=n\sigma_b$.

These statistical parameters can also be expressed in terms of vectors. Consider the vector of individuals' ideals, $\underline{x}$. The mean ideal is $\mu_x=\underline{x}.\underline{1}/n$. Writing $\underline{\mu}_x=\mu_x.\underline{1}$ and $(\underline{x}-\underline{\mu}_x)=\underline{d}_x$, the variance is $\sigma_x^2=||(\underline{x}-\underline{\mu}_x)||^2/n=||\underline{d}_x||^2/n$ and so $\sigma_x=||\underline{d}_x||/\sqrt{n}$. The vector of biases is $\underline{b}$ with $||\underline{b}||=\sigma_b\sqrt{n}$.

THEOREM 6. *The deviation D of the social outcome from optimality depends on: the bias magnitude b^; the variation σ_x^2 in individuals' ideals; and the correlation ρ_{xb} between the bias vector and the vector of ideals.*

$$D = n\,\sigma_b\,\sigma_x\,\rho_{xb} = b^\wedge\,\sigma_x\,\rho_{xb} = ||\underline{b}||\;||\underline{d}_x||\;\rho_{xb} \quad (4.1)$$

PROOF
$D = x\text{-}x^* = \Sigma w'_i x_i - \Sigma w_i x_i = \Sigma(w_i - w'_i)x_i = \Sigma b_i x_i = n\,\text{cov}(b_i, x_i) = n\,\sigma_b\,\sigma_x\,\rho_{xb}$

THEOREM 7. *Suppose the set of social states is multi-dimensional. The deviation $\underline{D}$ of the social outcome from optimality is then a vector. The norm of $\underline{D}$ depends on the bias magnitude $b^\wedge$; on the variation in individuals' ideals s_x^2; and on R^2, a variation-weighted mean of the squares of the correlation between the bias vector and the vector of ideals ρ_{xb}, $||D||=bs_xR$.*

PROOF
$\underline{D} = \underline{x}\text{-}\underline{x}^*$. The j component of this vector is $D_j = b^\wedge\,s_{xj}\,\rho_{xjb}$.
$||D||^2 = \Sigma_j D_j^2 = \Sigma_j (b^\wedge\,s_{xj}\,\rho_{xjb})^2 = b^{\wedge 2}\,s_x^2\,\Sigma_j ((s_{xj}/s_x)^2\;\rho_{xjb}^2) = b^{\wedge 2}\,s_x^2\,R^2$
where $R^2 = \Sigma_j ((s_{xj}/s_x)^2\;\rho_{xjb}^2)$ and $s_x^2 = \Sigma_j s_{xj}^2$.

5. Incorporating social ideals into the social process … errors

The social welfare function is often seen as standing outside the social process. Thus in the model so far the welfare ideal does not enter into the equation for the social outcome. The only ideals to take part in the process are the individuals' own personal ideals. We now allow for the possibility that an individual may exert their power in pursuit of some broader purpose, some social ideal which can give others' personal ideals non-zero weight (this embraces the pursuit of a purely personal ideal as a special case) reflecting their 'social preference' (Fehr and Fischbacher, 2002; Charness and Rabin, 2002, pp. 851-858). So we define the social ideal x^*_i for each individual i as some weighted sum of the personal ideals of the population: $x^*_i=\Sigma_k w'_{ik}x_k$. The weight w'_{ik} constitutes the importance which i attaches to k in i's social ideal. Each individual i exerts their direct power w_i' in pursuit of their social ideal x^*_i. One component of this direct power is $w_i'w'_{ik}$ which is the contribution which i makes to the pressure for k's personal ideal. The pressure for the ideal of k, in other words the indirect power of k, w'_k, consists of the sum of these contributions $w'_k=\Sigma w_i'w'_{ik}$.

THEOREM 8.
The social outcome x' is the weighted sum of social ideals, with the weights being the direct power of the individuals. The social outcome can also be expressed as the weighted sum of personal ideals, with the weights being the indirect power (that is the sum of contributions from other person's social ideals, weighted by the other persons' direct power).

$$x' = \Sigma_i\,w_i'\,x^*_i = \Sigma_k\,w_k'x_k \quad (5.1)$$

PROOF
$x' = \Sigma_I\,w_i'x^*_I = \Sigma_I\,w_i'(\Sigma_k\,w_{ik}'x_k) = \Sigma_k\,(\Sigma_I\,w_i'w_{ik}')\,x_k = \Sigma_k\,w_k'x_k$

Is society better off if individuals pursue social ideals rather than simply their own personal ideals? Not necessarily. Any outcome in the convex hull of personal ideals can be achieved by a suitable pattern of social ideals. Some social ideals bring the outcome closer to the welfare ideal, other social ideals are neutral while other social ideals take the outcome further away from the ideal. There is an analogy here with the notion of a tax as progressive, neutral or regressive. Not only can any outcome be achieved by social ideals, the same social outcome can be achieved by many different patterns of social ideals. For example we can achieve the welfare ideal either by each individual adopting the welfare ideal as their social ideal; or by the powerful giving suitable compensation to the weak.

The matrix $W=[w_{ij}]$ of contributions represents the network of social linkages which are implicit in the set of social ideals. If the population is large then the social matrix is likely to be quite sparse – each individual can only consider a few other individuals. If the linkages occur at random then the effect of social ideals will be neutral. If the linkages are 'assortative' (to use terminology from evolutionary theory) then the amount of bias may be maintained.

An individual may be mistaken as to what their ideal is, or their efforts might have the effect of supporting an outcome different from their ideal. The aggregate error is the sum of the individual errors. The error can be progressive, neutral or regressive. To summarise, the deviation of the social outcome from the welfare ideal is the sum of the personal ideal outcome, the social ideal addition and the error addition – minus the welfare ideal.

6. Groups

In the model so far the population of individuals is undifferentiated. Yet many conflict situations involve a group structure with conflict not only between groups but also conflict within groups. Theorem 10 below shows that the deviation for the population is the sum of the within-group deviation and the between-group deviation. First we note how any population attribute can be expressed in terms of the attributes of the constituent groups.

THEOREM 9. *Consider a population attribute z which is a weighted sum of individual attributes z_i. Suppose the population is divided into two distinct groups, G and H. We define the relative powers of G and H, g and h respectively, with respect to z as $g=\Sigma_g w_i$ and $h=\Sigma_h w_i$. So $g+h=1$. We define the within-group weights: $w_{ig}=w_i/g$, and $w_{ih}=w_i/h$. We define the subgroup attributes: $z_g=\Sigma_g w_{ig} z_i$ and $z_h=\Sigma h_h w_{ih} z_i$. Then $z=gz_g+hz_h$.*

PROOF
$z = \Sigma w_i z_i = \Sigma_g w_i z_i + \Sigma_h w_i z_i = g\Sigma_g w_{ig} z_i + h\Sigma_h w_{ih} z_i = gz_g + bz_h$

THEOREM 10. *The deviation for the population is the sum of the within-group deviation D_W and the between-group deviation D_B. The former is the weighted sum of the deviations within each group, D_g and D_h. The latter is the between-group bias b_g times the difference between the group ideals.*

$$D = D_W + D_B \qquad D_W = gD_g + hD_h \qquad D_B = b_g(x^*_g - x^*_h)$$

PROOF
$D = x - x^* = (gx_g + hx_h) - (g^*x^*_g + h^*x^*_h)$
Consider $gx_g - g^*x^*_g = gx_g - gx^*_g + gx^*_g - g^*x^*_g = g(x_g - x^*_g) + (g - g^*)z'^*_g = gD_g + b_g x^*_g$
So $D = gD_g + hD_h + b_g(x^*_g - x^*_h)$ because $b_g = -b_h$

7. Conclusion

In Sections 2 and 3 some quite severe restrictions were made and this enabled us to express both the social welfare and the social outcome as a weighted sum of individual ideals and to develop a number of results. Restricted though it is the model has the merit of encapsulating some of the key ideas on social processes and social optimality. One application has been the identification of the welfare ideal and the estimation of the welfare loss incurred by a given situation (Burt, 2004). Also, Stokman and Zeggelink (1996) provide an application of the power-weighted model.

In general any model of a conflict situation will offer one or more ‘solution concepts’. A solution concept either predicts what will happen (interaction solution concepts) or identifies certain outcomes which meet specified criteria (value solution concepts). In this article the value solution concepts have been the social welfare maximum, the convex hull of personal ideals, the welfare ideal, and the Pareto optimum; and the interaction solution concept has been the power-weighted ideal. A quite different model of the social process is that provided by game theory, the interaction solution concept there being the Nash equilibrium. Moreover game theory applications involve utility functions which may possess none of the special properties discussed in the present article. The game theory literature reminds us that in general the social outcome may not correspond to any of the solution concepts and moreover none of the solution concepts may correspond to any of the other solution concepts.

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