The Maths inside...
CLEVER

Narrow cars may take up less space but they have a problem - they tend to roll as they go around corners, and can even roll over if they go round the corner too fast. When a bicycle or a Pendolino train goes round a corner, it tilts into the corner. So the CLEVER team developed a tilting mechanism to make cornering possible using the branch of mathematics called **dynamics**.

When the car goes round a corner at a speed V it has to accelerate towards the centre of corner – this happens by the car's tyres creating a sideways force into the corner. The acceleration is given by

$$a_{corner} = \frac{V^2}{R}$$

where R is the radius of the corner. The tighter the corner is, the greater the acceleration is. But if this acceleration is too great, the weight lifts off the inside wheel and the car will roll over.

The researchers had to find the greatest acceleration that the car could cope with without rolling over. This is given by the equation

$$a_{\max} = \frac{gt}{2h}$$

where t is the width of the car, h is the height of the centre of gravity of the car and g is the acceleration due to gravity.

The relationship between the width of the car and the height of its centre of gravity is crucial to determining the maximum acceleration round a corner. Since CLEVER is only 1m wide, the team needed to ensure that the height of the centre of gravity h is also small, otherwise the car would always have to go very slowly to get around corners safely.

They discovered that h needed to be maximum 0.5m. But they did not want the occupants to be sitting so low, as this would be problematic for getting in and out of the car and also make visibility difficult.

So they developed a tilting mechanism which leans the body of the vehicle like a motorbike at angles upto 45°. This shifts the centre of gravity sideways and pushes the weight on to the inside wheel thus avoiding roll-over. In fact, CLEVER is very clever, and uses a computer control system to work out the best tilt angle, which depends on the speed V of the car and also on the angle at which the driver steers into the corner as this changes the corner radius, R.

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